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Teachers' Manual to Walsh's Primary Arithmetic.

MATHEMATICS FOR COMMON SCHOOLS

A

MANUAL FOR TEACHERS

INCLUDING

DEFINITIONS, PRINCIPLES, AND RULES
AND SOLUTIONS OF THE MORE
DIFFICULT PROBLEMS

BY

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ASSOCIATE SUPERINTENDENT OF PUBLIC INSTRUCTION
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PRIMARY ARITHMETIC

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ANSWERS 1

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MANUAL FOR TEACHERS

I

INTRODUCTORY

Plan and Scope of the Work. — In addition to the subjects generally included in the ordinary text-books in arithmetic, *Mathematics for Common Schools* contains such simple work in algebraic equations and constructive geometry as can be studied to advantage by pupils of the elementary schools.

The arithmetical portion is divided into thirteen chapters, each of which, except the first, contains the work of a term of five months. The following extracts from the table of contents will show the arrangement of topics:

First and Second Years

Chapter I. — Numbers of Three Figures. Addition and Subtraction.

Third Year

Chapters II. and III. — Numbers of Five Figures. Multipliers and Divisors of One Figure. Addition and Subtraction of Halves, of Fourths, of Thirds. Multiplication by Mixed Numbers. Pint, Quart, and Gallon; Ounce and Pound. Roman Notation.

Fourth Year

Chapters IV. and V. — Numbers of Six Figures. Multipliers and Divisors of Two or More Figures. Addition and Subtraction of Easy Fractions. Multiplication by Mixed Numbers. Simple Denominate Numbers. Roman Notation.

Fifth Year

Chapters VI. and VII. — Fractions. Decimals of Three Places. Bills. Denominate Numbers. Simple Measurements.

Sixth Year

Chapters VIII. and IX. — Decimals. Bills. Denominate Numbers. Surfaces and Volumes. Percentage and Interest.

Seventh Year

Chapters XI. and XII. — Percentage and Interest. Commercial and Bank Discount. Cause and Effect. Partnership. Bonds and Stocks. Exchange. Longitude and Time. Surfaces and Volumes.

Eighth Year

Chapters XIII. and XIV. — Partial Payments. Equation of Payments. Annual Interest. Metric System. Evolution and Involution. Surfaces and Volumes.

While all of the above topics are generally included in an eight years' course, it may be considered advisable to omit some of them, and to take up, instead, during the seventh and eighth years, the constructive geometry work of Chapter XVI. Among the topics that may be dropped without injury to the pupil are Bonds and Stocks, Exchange, Partial Payments, and Equation of Payments.

Grammar School Algebra. — Chapter X., consisting of a dozen pages, is devoted to the subject of easy equations of one unknown quantity, as a preliminary to the employment of the equation in so much of the subsequent work in arithmetic as is rendered more simple by this mode of treatment. To teachers desirous of dispensing with rules, sample solutions of type examples, etc., the algebraic method of solving the so-called "problems" in percentage, interest, discount, etc., is strongly recommended.

In Chapter XV., intended chiefly for schools having a nine years' course, the algebraic work is extended to cover simple equations containing two or more unknown quantities, and pure and affected quadratic equations of one unknown quantity.

No attempt has been made in these two chapters to treat algebra as a science; the aim has been to make grammar-school pupils acquainted, to some slight extent, with the great instrument of mathematical investigation, — the equation.

Constructive Geometry. — Progressive teachers will appreciate the importance of supplementing the concrete geometrical instruction now given in the drawing and mensuration work. Chapter XVI. contains a series of problems in construction so arranged as to enable pupils to obtain for themselves a working knowledge of all the most important facts of geometry. Applications of the facts thus ascertained, are made to the mensuration of surfaces and volumes, the calculation of heights and distances, etc. No attempt is made to anticipate the work of the high-school by teaching geometry as a science.

While the construction problems are brought together into a single chapter at the end of the book, it is not intended that instruction in geometry should be delayed until the preceding work is completed. Chapter XVI. should be commenced not later than the seventh year, and should be continued throughout the remainder of the grammar-school course. For the earlier years, suitable exercises in the mensuration of the surfaces of triangles and quadrilaterals, and of the volumes of right parallelopipedons have been incorporated with the arithmetic work.

II

GENERAL HINTS

Division of the Work. — The five chapters constituting Part I. of *Mathematics for Common Schools* should be completed by the end of the fourth school year. The remaining eight arithmetic chapters constitute half-yearly divisions for the second four years of school. Chapter I., with the additional oral work needed in the case of young pupils, will occupy about two years; the remaining four chapters should not take more than half a year each. When the Grube system is used, and the work of the first two years is exclusively oral, it will be possible, by omitting much of the easier portions of the first two chapters, to cover, during the third year, the ground contained in Chapters I., II., and III.

Additions and Omissions. — The teacher should freely supplement the work of the text-book when she finds it necessary to do so; and she should not hesitate to leave a topic that her pupils fully understand, even though they may not have worked all the examples given in connection therewith. A very large number of exercises is necessary for such pupils as can devote a half-year to the study of the matter furnished in each chapter. In the case of pupils of greater maturity, it will be possible to make more rapid progress by passing to the next topic as soon as the previous work is fairly well understood.

Oral and Written Work. — The heading "Slate Problems" is merely a general direction, and it should be disregarded by the teacher when the pupils are able to do the work "mentally." The use of the pencil should be demanded only so far as it may

be required. It is a pedagogical mistake to insist that all of the pupils of a class should set down a number of figures that are not needed by the brighter ones. As an occasional exercise, it may be advisable to have scholars give all the work required to solve a problem, and to make a written explanation of each step in the solution; but it should be the teacher's aim to have the majority of the examples done with as great rapidity as is consistent with absolute correctness. It will be found that, as a rule, the quickest workers are the most accurate.

Many of the slate problems can be treated by some classes as "sight" examples, each pupil reading the question for himself from the book, and writing the answer at a given signal without putting down any of the work.

Use of Books. — It is generally recommended that books be placed in pupils' hands as early as the third school year. Since many children are unable at this stage to read with sufficient intelligence to understand the terms of a problem, this work should be done under the teacher's direction, the latter reading the questions while the pupils follow from their books. In later years, the problems should be solved by the pupils from the books with practically no assistance whatever from the teacher.

Conduct of the Recitation. — Many thoughtful educators consider it advisable to divide an arithmetic class into two sections, for some purposes, even where its members are nearly equal in attainments. The members of one division of such a class may work examples from their books while the others write the answers to oral problems given by the teacher, etc.

Where a class is thus taught in two divisions, the members of each should sit in alternate rows, extending from the front of the room to the rear. Seated in this way, a pupil is doing a different kind of work from those on the right and the left, and he would not have the temptation of a neighbor's slate to lead him to compare answers.

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As an economy of time, explanations of new subjects might be given to the whole class; but much of the arithmetic work should be done in "sections," one of which is under the immediate direction of the teacher, the other being employed in "seat" work. In the case of pupils of the more advanced classes, "seat" work should consist largely of "problems" solved without assistance. Especial pains have been taken to so grade the problems as to have none beyond the capacity of the average pupil that is willing to try to understand its terms. It is not necessary that all the members of a division should work the same problems at a given time, nor the same number of problems, nor that a new topic should be postponed until all of the previous problems have been solved.

Whenever it is possible, all of the members of the division working under the teacher's immediate direction should take part in all the work done. In mental arithmetic, for instance, while only a few may be called upon for explanations, all of the pupils should write the answers to each question. The same is true of much of the sight work, the approximations, some of the special drills, etc.

Drills and Sight Work. — To secure reasonable rapidity, it is necessary to have regular systematic drills. They should be employed daily, if possible, in the earlier years, but should never last longer than five or ten minutes. Various kinds are suggested, such as sight addition drills, in Arts. 3, 11, 24, 26, etc.; subtraction, in Arts. 19, 50, 53, etc.; multiplication, in Arts. 71, 109, etc.; division, in Arts. 199, 202, etc.; counting by 2's, 3's, etc., in Art. 61; carrying, in Art. 53, etc. For the young pupil, those are the most valuable in which the figures are in his sight, and in the position they occupy in an example; see Arts. 3, 34, 164, etc.

Many teachers prepare cards, each of which contains one of the combinations taught in their respective grades. Showing one of these cards, the teacher requires an immediate answer

from a pupil. If his reply is correct, a new card is shown to the next pupil, and so on. Other teachers write a number of combinations on the blackboard, and point to them at random, requiring prompt answers. When drills remain on the board for any considerable time, some children learn to know the results of a combination by its location on the board, so that frequent changes in the arrangement of the drills are, therefore, advisable. The drills in Arts. 111, 112, and 115 furnish a great deal of work with the occasional change of a single figure.

For the higher classes, each chapter contains appropriate drills, which are subsequently used in oral problems. It happens only too frequently that as children go forward in school they lose much of the readiness in oral and written work they possessed in the lower grades, owing to the neglect of their teachers to continue to require quick, accurate review work in the operations previously taught. These special drills follow the plan of the combinations of the earlier chapters, but gradually grow more difficult. They should first be used as sight exercises, either from the books or from the blackboard.

To secure valuable results from drill exercises, the utmost possible promptness in answers should be insisted upon.

Definitions, Principles, and Rules. — Young children should not memorize rules or definitions. They should learn to add by adding, after being first shown by the teacher how to perform the operation. Those not previously taught by the Grube method should be given no reason for "carrying." In teaching such children to write numbers of two or three figures, there is nothing gained by discussing the local value of the digits. During the earlier years, instruction in the art of arithmetic should be given with the least possible amount of science. While principles may be incidentally brought to the view of the children at times, there should be no cross-examination thereon. It may be shown, for instance, that subtraction is the reverse of addition, and that multiplication is a short method of combining equal

numbers, etc.; but care should be taken in the case of pupils below about the fifth school year not to dwell long on this side of the instruction. By that time, pupils should be able to add, subtract, multiply, and divide whole numbers; to add and subtract simple mixed numbers, and to use a mixed number as a multiplier or a multiplicand; to solve easy problems, with small numbers, involving the foregoing operations and others containing the more commonly used denominate units. Whether or not they can explain the principles underlying the operations is of next to no importance, if they can do the work with reasonable accuracy and rapidity.

When decimal fractions are taken up, the principles of Arabic notation should be developed; and about the same time, or somewhat later, the principles upon which are founded the operations in the fundamental processes, can be briefly discussed.

Definitions should in all cases be made by the pupils, their mistakes being brought out by the teacher through appropriate questions, criticisms, etc. Systematic work under this head should be deferred until at least the seventh year.

The use of unnecessary rules in the higher grades is to be deprecated. When, for instance, a pupil understands that *per cent* means *hundredths*, that seven per cent means seven hundredths, it should not be necessary to tell him that 7 per cent of 143 is obtained by multiplying 143 by .07. It should be a fair assumption that his previous work in the multiplication of common and of decimal fractions has enabled him to see that 7 per cent of 143 is $\frac{7}{100}$ of 143 or $143 \times .07$, without information other than the meaning of the term "per cent."

When a pupil is able to calculate that 15 % of 120 is 18, he should be allowed to try to work out for himself, without a rule, the solution of this problem: 18 is what per cent of 120? or of this: 18 is 15 % of what number? These questions should present no more difficulty in the seventh year than the following examples in the fifth: (*a*) Find the cost of $\frac{3}{20}$ ton of hay at $12 per ton. (*b*) When hay is worth $12 per ton, what part of a

ton can be bought for $1.80? (*c*) If $\frac{3}{20}$ ton of hay costs $1.80, what is the value of a ton?

When, however, it becomes necessary to assist pupils in the solution of problems of this class, it is more profitable to furnish them with a general method by the use of the equation, than with any special plan suited only to the type under immediate discussion.

In the supplement to the Manual will be found the usual definitions, principles, and rules, for the teacher to use in such a way as her experience shows to be best for her pupils. The rules given are based somewhat on the older methods, rather than on those recommended by the author. He would prefer to omit entirely those relating to percentage, interest, and the like as being unnecessary, but that they are called for by many successful teachers, who prefer to continue the use of methods which they have found to produce satisfactory results.

Language. — While the use of correct language should be insisted upon in all lessons, children should not be required in arithmetic to give all answers in "complete sentences." Especially in the drills, it is important that the results be expressed in the fewest possible words.

Analyses. — Sparing use of analyses is recommended for beginners. If a pupil solves a problem correctly, the natural inference should be that his method is correct, even if he be unable to state it in words. When a pupil gives the analysis of a problem, he should be permitted to express himself in his own way. Set forms should not be used under any circumstances.

Objective Illustrations. — The chief reason for the use of objects in the study of arithmetic is to enable pupils to work without them. While counters, weights and measures, diagrams, or the like are necessary at the beginning of some topics, it is important to discontinue their use as soon as the scholar is able to proceed without their aid.

Approximate Answers. — An important drill is furnished in the "approximations." (See Arts. 521, 669, 719, etc.) Pupils should be required in much of their written work to estimate the result before beginning to solve a problem with the pencil. Besides preventing an absurd answer, this practice will also have the effect of causing a pupil to see what processes are necessary. In too many instances, work is commenced upon a problem before the conditions are grasped by the youthful scholar; which will be less likely to occur in the case of one who has carefully "estimated" the answer. The pupil will frequently find, also, that he can obtain the correct result without using his pencil at all.

Indicating Operations. — It is a good practice to require pupils to indicate by signs all of the processes necessary to the solution of a problem, before performing any of the operations. This frequently enables a scholar to shorten his work by cancellation, etc. In the case of problems whose solution requires tedious processes, some teachers do not require their pupils to do more than to indicate the operations. It is to be feared that much of the lack of facility in adding, multiplying, etc., found in the pupils of the higher classes is due to this desire to make work pleasant. Instead of becoming more expert in the fundamental operations, scholars in their eighth year frequently add, subtract, multiply, and divide more slowly and less accurately than in their fourth year of school.

Paper *vs.* Slates. — To the use of slates may be traced very much of the poor work now done in arithmetic. A child that finds the sum of two or more numbers by drawing on his slate the number of strokes represented by each, and then counting the total, will have to adopt some other method if his work is done on material that does not permit the easy obliteration of the tell-tale marks. When the teacher has an opportunity to see the number of attempts made by some of her pupils to obtain the correct quo-

tient figures in a long division example, she may realize the importance of such drills as will enable them to arrive more readily at the correct result.

The unnecessary work now done by many pupils will be very much lessened if they find themselves compelled to dispense with the "rubbing out" they have an opportunity to indulge in when slates are employed. The additional expense caused by the introduction of paper will almost inevitably lead to better results in arithmetic. The arrangement of the work will be looked after; pupils will not be required, nor will they be permitted, to waste material in writing out the operations that can be performed mentally; the least common denominator will be determined by inspection; problems will be shortened by the greater use of cancellation, etc., etc. Better writing of figures and neater arrangement of problems will be likely to accompany the use of material that will be kept by the teacher for the inspection of the school authorities. The endless writing of tables and the long, tedious examples now given to keep troublesome pupils from bothering a teacher that wishes to write up her records, will, to some extent, be discontinued when slates are no longer used.

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III

EARLY ARITHMETIC TEACHING

Counting. — While the majority of children are able, upon entering school, to repeat the names of the first ten or more numbers, they are not always able to count things. The first duty of the teacher is to secure correct notions of the first nine numbers, and this can best be done by the employment of objects, such as beans, splints, shoe-pegs, blocks, etc. A numeral frame is very useful for this purpose.

In counting, it is very important to have the child understand that the *second* splint is not *two* splints. This may be made clear to a child by having him put on his desk one bean, then near it two beans, three beans in another place, etc. After the pupil can count understandingly to nine, he should be taught the figures. The notation and numeration of numbers of two or more figures will be discussed in later chapters.

Primary Arithmetic. — After children have learned to count readily, experts disagree as to the best method of procedure. Many excellent teachers believe that work should be commenced at once upon numeration and notation, followed by the fundamental operations in the usual order. Some of the advocates of this method favor the completion of each topic before proceeding to the next; that is, numeration and notation are taught at least to billions; then addition is taken up, beginning with small numbers and gradually increasing to examples containing numbers of eight or nine figures. Subtraction, multiplication, and division are each studied to this extent before the next is commenced.

The more intelligent advocates of teaching operations at the

outset, recognize the fact that it is neither necessary nor advisable to defer the addition of small numbers until children are able to write those of three or more periods, nor to postpone finding the sum of $\frac{1}{2}$ and $\frac{1}{4}$ until after the properties of numbers have been studied in the fifth school year. Their plan is to follow such simple examples in the addition of small numbers as involve no carrying, by corresponding ones in subtraction. More difficult examples in both of these operations come next, followed by simple ones in multiplication and division. Easy work in fractions is introduced at an early stage, and problems involving the more common denominate units are brought in from time to time.

The Grube Method. — A growing number of educators believe that early arithmetical instruction should be based upon the study of numbers, rather than upon that of processes, — that the former should be the prominent feature of the early instruction, and the latter incidental, at least for the first two years.

This method, called after its inventor, Grube, requires the teaching of all of the processes in the case of each number before proceeding to the next. Thus, when the number 4 is studied, the pupil measures it by all numbers smaller than itself. Using 4 beans, he measures by 1, by arranging them as follows: 0 0 0 0. In this way he sees that $1+1+1+1=4$; that there are 4 ones in 4, or $1 \times 4 = 4$; that $4-1-1-1=1$; that $4 \div 1 = 4$.

Measuring by 2, 00 00, he sees that $2+2=4$, $2 \times 2 = 4$, $4-2=2$, $4 \div 2 = 2$.

Measuring by 3, 000 0, he sees that $3+1=4$, $1+3=4$; $4-3=1$, $4-1=3$; that $(1 \times 3)+1=4$, and that $4 \div 3 = 1$ and 1 over.

The pupil then answers questions given by the teacher, first using the counters and afterwards without them: —

Four is how many more than 3? Than 1? Than 2? Three is how many less than 4? Two is how many less? One is how many less?

How many ones in 4? How many twos? Threes? One-half of 4 is what? Two is $\frac{1}{2}$ of what number?

Problems containing the foregoing combinations are then given in great variety by the teacher until all of the facts about the number 4 in its relation with the smaller numbers are fully mastered.

In teaching any number, no larger number must appear in any way whatever. During the study of 4, it is not permissible to ask 4 twos, or that 4 is 1 less than what, etc., etc.

The work proceeds slowly and thoroughly, at least a year being devoted to the numbers below 10. The second year is given to the numbers from 10 to 20, and the third year to those from 20 to 100. This is probably as far as the method is carried in this country.

In the greater number of the schools using this method, systematic instruction in the fundamental processes is commenced by the beginning of the third year; while in some, the Grube method is used for oral work, and the teaching of slate addition is carried on at the same time, even during the first year.

Slate Problems. — When, instead of receiving oral instruction for some time, children are taught processes from the outset, it frequently happens that many of them show little ability in solving problems. While some attention should be given in the early years to this side of arithmetic, it should not be permitted to retard too much the advancement of pupils. Many of them have to leave school soon, and they should be taught as rapidly as is consistent with real progress to perform accurately the ordinary operations in whole numbers, simple fractions, and decimals. Being familiar with these tools, greater maturity will, of itself, show which is to be used in such questions as are likely to come up in ordinary avocations.

The teacher should exercise much care to give only such problems as can readily be understood by the pupil, and which do not contain too many conditions or numbers that bewilder

the learner. While a beginner will have no difficulty in determining whether to add or subtract in a mental problem suited to his capacity, the same kind of problem with larger figures will give him much difficulty. For this reason, the earlier slate problems should be the merest trifle beyond his ability to solve mentally. In his attempt to work them out in his head, he will determine whether addition or subtraction is needed, etc.

Problems in all grades should be "miscellaneous," and pupils should be allowed as far as possible to determine for themselves what operation is necessary to solve any given one.

IV

NOTES ON CHAPTER ONE

THE hints given as to the work of this chapter are intended chiefly for the guidance of teachers of young children that are beginning slate work in the fundamental processes without much preliminary oral instruction. Pupils that have been taught for two years by the Grube method should not be required to spend unnecessary time on the simpler portions of the work.

Art. **4.** — In teaching notation of numbers of two figures to young children that have not been previously taught by the Grube method, it is not advisable to lay stress on the local value of the tens' figure. Show them how to read and write 10, 11, 12, etc., to 20; then 30, 40, 50, etc., to 90. After this, there is but little difficulty.

7. By working an example for the pupils, teach them to place under each column its sum. As their tendency is to begin working at the left, be careful to see that they always commence to add at the right.

9. The problems will present no difficulty, as they involve only addition.

11. These sight exercises may first be employed as drills to teach children to use in blackboard addition as few words as possible. The first figure should not be named, — only the sum of the first and the second, then this total added to the third. In subsequent drills upon these combinations, each pupil should, in turn, give the sum of any set indicated by the teacher. The work should be done rapidly to be of value.

13. The making of original problems by the pupils should be a feature of every grade.

15 and **16.** Subtraction is here introduced by the "building-up" method. Pupils find it easier to ascertain the difference between two numbers by going forward from the smaller to the larger, than by "taking away" one from the other.

17 consists of sight exercises in the form of addition, leading to the subtraction exercises in Art. 19.

21. While in adding, the use of the word *and* is considered unnecessary; in subtracting, it is used just before the figure that is to be written.

For some advantages obtained by employing the "building-up" method, see Art. 384, where it is used to obtain in one operation the difference between 1000 and $643 + 287 + 25$. In Art. 385, it is used to find a remainder in long division without writing the product of the divisor by the quotient.

23. Here begins the real problem work, as the pupil has now to determine for the first time in slate examples whether the result is to be reached by addition or subtraction. When the pupils are able to solve one of these problems without using the pencil, it should be repeated, but with such a change in one of the numbers as will render necessary the use of the slate. For the 10 cents in the first example, for instance, 14 cents or 24 cents may be substituted.

As many pupils attend rather to the numbers in a problem than to its terms, some may subtract when they should add, especially as this seems the natural operation when only two numbers are involved. It is important that they should be led to see that the size of the numbers does not change the nature of the example, and that they can easily determine whether addition or subtraction is required, by considering what operation

they would employ in a similar example containing very small figures.

It is not advisable as a regular thing to follow an oral problem by a written one of exactly the same nature, as this tends to make children inattentive to the terms of the latter when they already know from the oral problem what operation is required.

28. It is inadvisable to waste time in endeavoring to make clear to very young children the reason for "carrying."

37. Teachers should require pupils to write the proper sign before working an example, as this tends to make them listen more carefully in order to determine whether addition or subtraction is involved. In some problems that are too simple to need the use of the pencil, changes may be made in the numbers employed; great care, however, should be taken not to use numbers so large as to confuse the pupils.

38. Have children understand that when a number contains the word "hundred," it should consist of three figures. Do not explain.

54. These exercises are intended to lead up to the subtraction with "borrowing" in the next article. Perhaps the following would be a better arrangement:

?	?	?	?	?	?
+29	+37	+17	+86	+ 75	+ 90
41	50	25	90	100	150

As children are generally taught to begin with the bottom figure in addition, they will naturally say in the first example, 9 and 2 are 11, writing the 2 in its place, etc.

55. Subtraction with "borrowing" is generally taught in one of three ways. The "building-up" method given in the text is the most readily taken hold of by young pupils.

$\begin{array}{r} {}^{11} \\ 41 \\ {}^{8} \\ 29 \\ \hline \end{array}$ By the second method, the child is instructed that whenever he increases by ten any figure of the minuend, he must add 1 to the next figure of the subtrahend. Seeing that he cannot take 9 from 1, he says 9 from 11 leaves 2; 1 (to carry) and 2 are 3, 3 from 4 leaves 1.

While this method is just as logical as the next, it is not so easily "explained," and, for this reason, is not so much favored by many teachers of the present day.

The third method consists of diminishing the next left-hand figure of the minuend after "borrowing." Where the minuend contains ciphers, this method is particularly confusing to beginners, especially where they are forbidden, as should be the case, to write the changes that are made in the figures of the minuend.

$\begin{array}{r} {}^{3}{}^{11} \\ 4\not{1} \\ 29 \\ \hline \end{array}$

Except in the addition of long columns, children should be required from the beginning of slate work to abstain from counting, writing "carrying" figures, and the like. The guide figures introduced into the foregoing explanations of methods of subtracting should not be used by pupils.

61. As a change from sight work, and to increase the pupils' readiness in the solution of mental examples, these drills are useful. Not requiring any preliminary writing on the board, they can be taken up at any time the class is unoccupied for a few minutes — waiting for the signal to go home, for example.

The pupils all stand; the teacher announces the number to be added, 2 for instance, and begins by saying 1 herself. The first pupil says 3, then sits; the next, 5; and so on. After 39, or some other convenient number, is reached, the teacher begins by saying 2, and the pupils, in order, give 4, 6, 8, etc., to 40.

The intelligent teacher will be careful to suit these drills to the capacity of her pupils. She will not weary beginners by spending too much time on the more difficult drills with 7, 8, and 9; nor will she waste the time of older scholars by dwelling on the addition by twos.

The same kind of work may be employed as subtraction drills.

Subtract by twos:

40, 38, 36, etc.
39, 37, 35, etc.

By threes:

40, 37, 34, etc.
39, 36, 33, etc.
38, 35, 32, etc.

By fours:

40, 36, 32, etc.
39, 35, 31, etc.
38, 34, 30, etc.
37, 33, 29, etc.

V

NOTES ON CHAPTER TWO

74. Slate multiplication is commenced as soon as the table of 2 times is learned. The first examples contain no carrying.

76. Division tables should not be memorized.

81. Do not permit children to prefix an unnecessary cipher in the quotient of $100 \div 2$; that is, do not have the answer written 050.

84. Many scholars think that when a slate problem contains a very small number and a large one, they must either multiply or divide. Examples 1–4 are given with simple numbers to show them that the nature of the operation depends entirely upon the conditions of a problem. While pupils should not be required to use a pencil to solve a problem that can be solved mentally, it would help the class to have these four examples worked on the board as an indication that in the subsequent examples there may be needed any one of the four operations learned thus far, and to serve as a model in their arrangement of the other problems.

While many teachers require the pupils to write the denomination of each addend, of the subtrahend and the minuend, of the multiplicand, and of the dividend, it is scarcely necessary. In later life it is not done; and confusion is sometimes produced in the minds of young scholars by attempting to make them understand why, for example, 60 pints divided by 2 will sometimes give a quotient of 30 pints, and at other times, as in the 6th

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problem, an apparent quotient of 30 quarts. It will be found more satisfactory, even if less scientific, to have the denomination written only with the result.

Although no formal instruction in finding halves and thirds of numbers has as yet been given, the average pupil will be able to solve problems 10, 11, and 14.

85. Lay no stress on the local value of the figures. Practice will enable the children to read and write correctly numbers of four figures. Teach the pupil to write the comma when the word "thousand" is said and after the number of thousands, the comma to be followed always by three figures.

97. Children should be led to see that 12×2 is the same as $12 + 12$; so that when they come to 15×2, they will have no difficulty in deducing the rule for writing 0 and carrying 1 when they multiply the 5 by 2.

98. Give the pupils time to find for themselves the quotient of $30 \div 2$. If it becomes necessary to show some of them how to work the example, do not elaborate the meaning of the 1 (ten) remainder when the tens' figure, 3, is divided by 2. An experienced mathematician, in dividing 9752 by 2, does not say 2 into 9 thousand 4 thousand times with a remainder of 1 thousand, 2 into 1700 8 hundred times with a remainder of 1 hundred, etc.

In dividing 30 by 2, children should not be permitted to write the first remainder, 1, before the 0, to indicate that 2 is to be divided into 10 for the second quotient figure. Children learn to work just as well without these unnecessary scaffolds.

104. While these drill exercises introduce a multiplier greater than 2, they contain no combinations, except 3×3, other than those found in the preceding work. After working these examples, the pupils will have learned that twice 9 is equal to 9 twos, — that when he knows the table of 2's, he knows a portion at least of the table of 3's, 4's, etc., to 9's.

111. When the teacher places the pointer on a number in one of the two outer spaces of the first circle, the pupil promptly gives the result obtained by adding to it the number contained in the inmost space. When this last number has been combined with all the others, it is replaced by a different number.

112. These drills are useful to impress upon a child the fact that when he knows, for instance, that 6 and 5 are 11, he should also know that 6 and 15 are 21, that 6 and 25 are 31, etc. They may also be employed as subtraction drills.

115. Division drills are necessary to enable pupils to acquire facility in obtaining quotients and remainders. When pupils are dividing by 2, the numbers from, say, 9 to 19 are written on the board with 2 underneath.

9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
÷ 2.

When the pointer is placed at the 9, the pupil answers 4 and 1; when placed at 14, he answers 7; at 17, 8 and 1; etc. Other divisors may be employed, but care should be taken not to have any quotient figure but 1 or 2 at this time, as pupils have not yet learned the table of 3's. Thus, when 6 is used as a divisor, the teacher should not use a dividend greater than 17. When the three-times table is known, numbers from 12 to 29 may be written.

Facility in division will come only by practice, and it may be necessary for the teacher to supplement the examples of the book by others of her own.

118. Do not fail to keep up practice in addition and subtraction.

119. Subtraction examples in which the subtrahend is given before the minuend should occasionally be used.

121. Do not worry a pupil by attempting to explain, through problem 9, the difference between division and partition. Let him write $\begin{array}{r} 2)\overline{50} \\ 25 \text{ pounds} \end{array}$ without taking advantage of the opportunity to show him that he should have an abstract quotient when the divisor and dividend are both concrete.

140. The analysis of problem 3 should not be required. A pupil that obtains from problem 2 the knowledge that 18 five-dollar bills amount to $90 will probably get the correct answer to the next problem, even though he may have to use 5 as a multiplier instead of the 18 that the more common form of the analysis would require. The other form should not be presented at this stage.

143. Children should be permitted to determine for themselves the method of obtaining the half of 36. It may require a little longer time than to show them, but the time will not be wasted.

147. Roman notation is not of much importance. Most children learn sufficient about it from the numbers affixed to their reading lessons.

157. Teachers should not endeavor to show by drawings that a quart measure is twice as large as a pint. If a pint measure is represented by a rectangle, each side of the rectangle indicating the quart should be only about $1\frac{1}{4}$ times that of the former in order to preserve the correct ratio, and children are not mathematicians enough to understand that where one of two similar solids has its corresponding dimensions $1\frac{1}{4}$ times those of the other, the volume of the former is double that of the latter. Use the measures themselves, borrowing them, if necessary, from a neighboring store.

159. A few problems involving more than one operation are here introduced. Avoid, if possible, giving help; and do not

require the scholars to perform unnecessary work, or to follow the same mode of solution or arrangement. In solving the first, some may write on their slates only two numbers, viz. 15 and 35. Others may set down 15, 15, and 20, etc. Do not teach yet how to multiply by a mixed number.

VI

NOTES ON CHAPTER THREE

While the teaching of formal definitions should find no place in the arithmetical instruction of the earlier years, the teacher should not hesitate to employ such technical terms as are called for by the work of the grade. Pupils gradually learn to understand what is meant by multiplier, quotient, remainder, etc., even where no attempt is made to explain the signification of the words. They will also become able to use each correctly, even if they cannot state its exact meaning in language that will satisfy a critical mathematician.

164. Sight exercises in division should be extended to cover dividends that are not multiples of the divisor. The slate examples in division supplied thus far have no remainders, as children find it more agreeable in the earlier stages of this work to have the answer a whole number. The partial dividends, however, do not always exactly contain the divisor, hence the need of such drills as will enable the pupil to determine rapidly the quotient figure and the remainder. Until Art. 176 is reached, this remainder need not be given by the pupils in the form of a fraction. See Art. 115.

168. In making "original problems," the pupil should strive to be original. No problem should be accepted as satisfactory that is substantially the same as one already furnished by another pupil. If, for example, the following is given to illustrate 12×5: "What will be the cost of 5 yards of ribbon at 12 cents a yard?" the teacher should not be satisfied with — "How

much will be paid for 5 pounds of cheese at the rate of 12 cents per pound?"

174. While the problems are gradually becoming more difficult, some of them can be done by bright pupils without using the pencil. In these cases, require that only the answers should be written. See previous notes on problem work. (Arts. 23, 84, and 159.)

178. Children should be permitted to follow their own plan of finding the product of 26 by $1\frac{1}{2}$. Some may do the work by simply placing 13 under 26. The regular method should not be taught until, perhaps, the 25th example, as the previous ones can be done by the children without assistance. At this point, however, the systematic way of multiplying by a mixed number may be presented, which should be followed in such subsequent examples as are not so simple as to make this amount of writing unnecessary, as is the case in the 26th.

$$\begin{array}{r} 26 \\ +13 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 124 \\ 2\frac{1}{4} \\ \hline 31 \\ 248 \\ \hline 279 \end{array}$$

In finding $\frac{1}{4}$ of 124, the pupil should not be permitted to write the multiplicand, 124, in some other part of his slate, and 4 as a divisor in front of it. No other writing of figures should be allowed than is given above. A little practice will enable scholars to perform this division and other similar operations, without always bringing into close contact the numbers to be handled.

In some European countries, the multiplier is placed at the right of the multiplicand, instead of being written underneath. An example like the 26th would be worked in that case without writing 760 a second time. To small children, however, it would be confusing to be required to learn two methods of working examples so nearly alike; hence the advisability of uniformly following the plan originally given, of first finding the fractional part, and then multiplying by the whole number.

$$760 \times 1\tfrac{1}{5}$$

$$\begin{array}{r} 760 \\ 1\frac{1}{5} \\ \hline 152 \\ 760 \\ \hline 912 \end{array}$$

180. The arrangement of work should begin to receive some attention. In solving the second problem, some children will find the cost of ½ pound of tea on one portion of the slate, and then write this amount, 35¢, on another part, with 25¢ underneath. They should be led to see how to avoid doing unnecessary work.

$$\begin{array}{r} 2\overline{)70} \\ 35 \\ +25 \\ \hline 60¢ \end{array}$$

186. Some short examples in the addition and the subtraction of horizontal columns are given, to accustom children to handle numbers that are not arranged for work in the usual way. The addition example could be used to explain the reason for "carrying," but the explanation should be deferred for the present.

191. Examples in division should occasionally be presented to pupils in the form used in the second column. When children recognize $\frac{27}{7}$ as an example in division, they need no rule for the reduction of an improper fraction to a whole or to a mixed number.

197. Do not furnish the pupils with a method of solving the 9th example that is suited to a sixth year class in denominate numbers. Leave them to their own resources as much as possible.

202. More drill examples are needed than are furnished in the book.

203. To secure good work in division, much practice must be given. Many more examples than are here supplied may be needed by some classes.

213. While it is convenient to write the subtrahend under the minuend, pupils should gradually accustom themselves to perform the fundamental operations with numbers in other than the usual positions.

$$\begin{array}{r} 7)119 \\ \hline 17 \\ \times 5 \\ \hline \text{etc.} \end{array}$$

215. Children should be encouraged to avoid unnecessary writing. They should be led to see that after finding on one part of the slate that $\frac{1}{7}$ of 119 is 17, they should not place this number in another place in order to multiply by 5.

220–223. These drills are intended to lead up to the use of larger numbers in the oral work of the pupils.

224. It is not advisable to begin formal instructions in fractions at this stage of school life. There is no need of defining the word "fraction" for the present. Every member of the class will be able to tell what is the sum of $\frac{1}{2}$ and $\frac{1}{2}$, especially if the question is put in the form of a problem.

It will be necessary, perhaps, to explain that $4 \times \frac{1}{2}$ is another way of expressing $\frac{1}{2}$ of 4; that $\frac{1}{2} \times 10$ means 10 halves. $1 \div \frac{1}{2}$ will also require translation into the form, "How many halves in 1?" Pupils may be led to see this by being asked to indicate by signs and figures the example, "How many twos are there in eighteen?" The drills in the use of fractional divisors need not be made prominent for the present.

230. Accustom children to writing the decimal point in the product, as soon as it is reached in multiplying. Reasons should not be dwelt upon.

231. The above applies to placing the decimal point in the quotient.

238. Unless pupils have been carefully trained to give only *reasonable* answers to slate problems, there will be some who will obtain 171 as the sum of $13\frac{1}{2}$ and $4\frac{1}{2}$. They will first write 1 as the equivalent of $\frac{1}{2}+\frac{1}{2}$; and to this they will prefix 17, obtained by adding 13 and 4. The special training in number received by pupils taught by the Grube method prevents to a great extent the absurd mistakes found in the answers

given by pupils, even of high-school classes, to simple problems. When the early arithmetical instruction is largely given to work in the fundamental processes, the teacher should make liberal use of oral problems, to give the requisite knowledge of number that will enable a pupil to know when his answer is very much out of the way. Systematic instruction in finding "approximate" results is supplied in later chapters.

239. These examples are intended to lead up to finding the difference between a whole number and a mixed number.

240. Pupils will find little difficulty in working out these examples if they are left to themselves.

241. When the addition and the subtraction of mixed numbers containing halves are readily performed, the teacher will find comparatively little trouble with the work under Arts. 241–245. Encourage pupils to make diagrams; or, if necessary, to divide circles into quarters, and to use these parts in performing the required operations with the fractions.

To find, for instance, the sum of $\frac{3}{4}+\frac{3}{4}$, it may be advisable to permit some scholars to arrange the six quarter-circles in such a way as to make a whole circle and a half-circle.

246. As children are more accustomed to dealing with halves and quarters than with thirds, a little more illustrative work may be needed in Arts. 246–250, than was required in the previous work in the addition and the subtraction of mixed numbers.

VII

NOTES ON CHAPTER FOUR

253–258. In the last chapter, pupils were required to add only fractions containing the same denominator; in this chapter, an addition or a subtraction example may contain fractions whose denominators are different. For the present, however, it will not be necessary to call attention to the need of reducing fractions to a common denominator. The average scholar can solve these examples without assistance, if he has been able to work out those found in Chapter III.

259. While these problems are becoming more difficult, they are still well within the powers of a pupil that is really anxious to solve them. When, however, they are found to be beyond the capacity of many members of the class, the teacher may first use them as "sight" problems, with some slight changes in the figures.

If, for instance, after a pupil that reads the first from his book declares that he is unable to obtain the answer mentally, the teacher may give it as follows:

A sailor has 10 yards of cloth. He uses 4 yards for a coat and 2 yards for a vest. How many yards has he left?

In the second, $1\frac{1}{2}$ pounds may be substituted for $1\frac{1}{4}$ pounds; in the third, 3 packages instead of 4; 20 dozen in the fourth, instead of $3\frac{1}{2}$ dozen.

Slate work on these problems should not be permitted until so many have been solved in this way that the pupil has had time to forget what operations have been used in each. This will

require him to study the conditions of the different problems, instead of relying upon his memory.

266. When the formal analysis of oral problems is made a feature of the work, it is important that the statements be not so long as to be tedious.

In the first, for example, the following would be sufficient, after the pupil has stated the problem:

"If 8 ounces of tea cost 40 cents, 1 ounce will cost 5 cents, and 5 ounces will cost 25 cents."

While the customary order has been followed in the systematic treatment of the various topics, pupils are called upon in the earlier chapters of *Mathematics for Common Schools* to solve many problems that are frequently deferred in other books to a later stage of their arithmetical instruction. While scholars readily solve this class of problems, they are not always able to state in technical language the reasons for the various processes employed in obtaining the answers. A child who sees that division is used to ascertain the number of ten-cent pies that can be purchased for forty cents, cannot be made to understand thus early in his school life that the same process is used to find what part of such a pie can be bought for five cents. A correct statement by the pupil of his method of reaching the result, should usually be accepted as satisfactory. Even in the more simple questions, set forms of analysis should be carefully avoided.

268. To prevent misunderstanding, parentheses have been employed even when not required by arithmetical usage. The quantities within the parentheses must be added, multiplied, etc., before being operated upon by the quantity outside. The third example becomes 30×3; the fourth, $80 \div 4$; the fifth, $\frac{1}{4}$ of 80; the eighth, $70 \div 7$, etc.

269. These may be used as slate examples, if they are found too difficult for "sight" work.

271. Some of these questions may not require the use of a pencil; Nos. 6, 7, 8, 11, and 19, for instance.

272. The answers to the first ten examples should be given at sight.

273. Use 49 to 57, inclusive, as "sight" examples; also as many as possible of those in the next section.

274. When the divisor ends in one or more ciphers, the latter are set off by a vertical bar, and also a corresponding number of figures from the right of the dividend. To keep the pupil from omitting these figures from the remainder, it is advisable to require him to write the partial remainder as above, before he begins to divide. Then, using 8 as a divisor, he writes the quotient figures in their places, and completes the partial remainder by prefixing 2 to the 1 that was originally brought down.

$8|0)\underline{434|1}$
$\frac{1}{80}$

$8|0)\underline{434|1}$
$54\frac{21}{80}$

It being the usual practice in abstract examples in division to refrain from reducing the fractional part of the quotient to lowest terms, the above method may be used in examples where both the divisor and the dividend terminate in a cipher. Some teachers prefer, however, in this case, to cancel the cipher in each, and to give the quotient of $4340 \div 80$ as $54\frac{3}{8}$.

$8|0)\underline{434|0}$
$54\frac{20}{80}$

$8\emptyset)\underline{434\emptyset}$
$54\frac{3}{8}$

277. Employ in "sight" work.

278. The foot-rule and the yardstick should be used by the children. They should ascertain, for instance, the length of their slates in inches, the length of the blackboard in yards or in feet, the height of the blackboard in feet, the dimensions of the room, etc.

280. It will be sufficient to accustom pupils to placing the product by the tens' figure one place to the left without giving the reason therefor. Neatness in the arrangement of the work, and the careful writing of figures, will prevent some mistakes.

282. In short division, the scholar has been taught to place the first figure of the quotient under the last figure of its partial dividend, and to write under each succeeding figure of the dividend its corresponding quotient figure. When his work is neatly arranged, he seldom omits ciphers, nor does he often obtain two quotient figures from one partial dividend.

$$4\overline{)12361}$$
$$3090\tfrac{1}{4}$$

To obtain the benefit of this experience, the pupil should be taught in long division to write the quotient over the dividend. By doing this, he will not be tempted, as are some beginners that place the quotient at the right, to give $23\frac{1}{21}$ as the answer to the above example; nor will he be likely to think that 252 contains 21, 111 times. This last result is obtained by assuming that the second partial dividend, 42, contains the divisor 1 time, with a remainder of 21. This latter is then made a partial dividend, with the above result.

$$\begin{array}{r} 2030\frac{1}{21} \\ 21\overline{)42631} \\ \underline{42} \\ 63 \\ \underline{63} \\ 1 \end{array}$$

285. While the pupil may write 16 as a multiplier in the 5th problem, he should be required to multiply by 30, in order to shorten the work. The multiplication by 30 should be performed, also, without rewriting the numbers so as to place 30 under 16.

$$\begin{array}{r} 30 \\ \times 16 \\ \hline \end{array}$$

286–290. The special drills will be found of great value in giving pupils a knowledge of numbers; and many oral problems employing these and similar combinations should be made by the teacher. Oral problems containing large numbers should, as a rule, require but one operation for their solution.

In the oral addition of numbers of two figures, the pupil should not commence, as in slate work, with the units' figures. The special drills of the last chapter should have taught him to think immediately of 80 when he sees $40+40$, $60+20$, $50+30$, etc. The next step in this work should contain such combinations as $47+40$, $63+20$, $54+30$, etc. In adding 54 and 30, the pupil should be taught to first see the *eighty*, then the *four*. The sum of 27 and 32 (the third step) should be obtained by joining 27 and 30 to make 57, and adding 2 to this result to obtain 59. If the pupil begins with the units, 7 and 2, he is likely to forget the tens' figures. When the addition work is readily performed, the pupil finds little trouble with the rest. There being no carrying, he will readily obtain the product of 32 by 3, and the others given in Art. 288, especially if he begins the multiplication at the tens' figure. After he becomes expert in adding and multiplying, he will experience no difficulty in subtracting and dividing.

294. The teacher should not encourage unnecessary work, by permitting children to write the sum of $12\frac{7}{8}+6\frac{1}{8}$ as $18\frac{8}{8}=18+1=19$. If, however, it be deemed advisable in the 4th example, for instance, that the fractions should be expressed with the same denominator, care must be taken to prevent pupils from making such mistakes as using the sign of equality between $\frac{1}{4}$ and $\frac{2}{8}$ in such a way as to represent that $50\frac{1}{4}$ is the equivalent of $\frac{2}{8}$. A vertical line drawn between the two sets of fractions will serve to separate the original example from the auxiliary portion. (See Art. 310.)

$$\begin{array}{l|l} 50\frac{1}{4} & \frac{2}{8} \\ 16\frac{3}{8} & \frac{3}{8} \\ \hline 66\frac{5}{8} & \frac{5}{8} \end{array}$$

304. As some children merely look for the figures of a problem without paying attention to its terms, an occasional one is given in which some or all of the numbers are expressed in words.

306. In making out a bill, it is convenient to be able to write the cost of 196 lb. at 4¢ per lb. without using another sheet of

paper and placing the 4 under 196. In working these examples, the pupil is expected to write only one figure of the product at a time. It is not intended that all of these twenty-five examples should be done before proceeding with the subsequent work. A few of them should be used from time to time throughout the term.

307. The last sentence applies also to these examples. A few of the easier ones should first be given. After more practice in long division, the more difficult ones may be taken up.

310. Whenever it becomes necessary, in the opinion of the teacher, to permit the rewriting of the fractions with a common denominator, she should, as soon as possible, have her pupils write the common denominator only once, as above. When the common denominator is written under each numerator, it is likely to be confusing to children, not to speak of the danger of its being added in occasionally with the numerators.

$$\begin{array}{r|l} & 8 \\ 49\frac{7}{8} & 7 \\ 20\frac{1}{2} & 4 \\ \hline 70\frac{3}{8} & \frac{11}{8} = 1\frac{3}{8} \end{array}$$

$$\begin{array}{r|l} 49\frac{7}{8} & \frac{7}{8} \\ 20\frac{1}{2} & \frac{4}{8} \\ \hline & \frac{11}{8} \end{array}$$

312. See Art. 268.

316. Where the multiplier ends with ciphers, some teachers think that time is saved by omitting the ciphers from the partial products. The ciphers at the right of the multiplier are written beyond the multiplicand, and are brought down at the end of the work.

$$\begin{array}{r} 76 \\ 1300 \\ \hline 228 \\ 76 \\ \hline 98800 \end{array}$$

Other teachers prefer to place the numbers as is generally done in multiplication, writing a cipher under each one in the multiplicand as its partial product, and writing the partial products by 3 and 1 under these figures, respectively. This method will be found to give more satisfactory results later on, when pupils have such multipliers as $20\frac{1}{2}$, $300\frac{1}{4}$, etc., in which a fraction follows the ciphers.

$$\begin{array}{r} 76 \\ 1300 \\ \hline 22800 \\ 76 \\ \hline 98800 \end{array}$$

319. See Art. 310. When an addition example consists of more than two mixed numbers with fractions of different denominators, it may be advisable to permit young children to write out the successive operations in the manner here indicated.

$$\begin{array}{r|l} & 6 \\ 5\frac{1}{2} & 3 \\ 7\frac{2}{3} & 4 \\ 9\frac{1}{6} & 1 \\ \hline 22\frac{1}{3} & \frac{3}{6} = 1\frac{2}{6} = 1\frac{1}{3} \end{array}$$

Many of the fifty examples on this page should be used as "sight" work from the blackboard, the pupils writing only the results. Nos. 1–6, 8–9, 13–16, 23–24, 26, 31–37, 41–43, 49–50 can be treated in this way after they have been worked out on the slate, if not in the first instance.

321. Until children obtain some knowledge of numbers, their progress in long division is very slow. In dividing 918 by 17, for instance, a pupil that is not properly instructed will sometimes take 1 as the first figure of the quotient. When, after subtracting, he obtains a remainder of 74, he may realize that he is wrong without being able to determine just how far astray he is. In this case he tries 2 as the quotient figure, ascertaining the product of 17×2 in a corner of his slate, and then transferring the 34 to its proper position under the first two figures of the dividend. Another subtraction follows, with a resulting remainder, again, perhaps, recognized as too great; and so on.

The object of these drills is to enable the scholar to reach at once a close approximation to the correct quotient figure. Their use may be commenced in some such way as the following:

The teacher writes on the blackboard a convenient number of those found among the first twenty, arranging them as shown below, with the divisor preceding the dividend. Under these she places the corresponding ones from the second and third sets, respectively.

20)160	60)360	90)450	50)300	30)270
19)160	59)360	89)450	49)300	29)270
21)160	61)360	91)450	51)300	31)270

Placing the pointer on those in the first row, successively, she receives the quotients promptly. She then asks for the quotient of the first in the third row, 21)160. If the pupil announces 8 as the result, he should be required to give mentally the product of 21×8, which he will find to be too great. He is thus led to see that the quotient is 7, with a remainder. The other quotients in this row are then elicited. After a pupil discovers that 21 is not contained 8 times in 160, that 61 is not contained 6 times in 360, etc., he may be introduced to the second row. A little questioning will enable him to perceive that if $160 \div 20 = 8$, the quotient of $160 \div 19$ must be at least 8, with a remainder; that $360 \div 59$ gives a quotient of 6, with something over, etc. Regular practice with this particular set of drills will rob division by 19, 29, 39, etc., of some of its terrors to slow pupils, as they will be led to use 2, 3, 4, etc., as "trial divisors" instead of 1, 2, 3, etc., whereby they will be able to obtain their answers in a reasonable time.

After the children have become able to announce at once the quotients of all the drills in the first three sets, and other similar ones supplied by the teacher, they may take up the remaining ones by degrees. When there is a remainder, the pupils should not be required to calculate it.

324. The quotient of $2{,}800 \div 200$ may be made more obvious if the dividend is read 28 hundred, instead of two thousand eight hundred.

328. See Art. 274, as to writing the partial remainder before beginning to divide.

341. Do not give reasons for the location of the partial products. There is plenty of time for the science of arithmetic later on in the course.

343. Although the divisors contain three or four figures, these examples should not prove so difficult as many of those already

worked. A pupil that has learned from the previous drills that $800 \div 200 = 4$, will be able to see that 201 is contained 4 times in the first three figures of 8,643. The teacher should be careful to see that the first quotient figure is written in its proper place.

$$\begin{array}{r} 4 \\ 201)8643 \\ 804 \\ \hline 603 \end{array}$$

etc.

No. 36 may cause some hesitation until the pupil perceives that he has to divide 81 hundred and something by 9 hundred and something. No. 37 will become simple if handled in the same way. In No. 47, 98 hundred divided by 12 hundred will give the clue to the quotient; in Nos. 48 and 50, nine thousand and two thousand should be used for this purpose.

344. With such a multiplier as 209, some teachers write a series of ciphers to denote the product by 0. The method given in the text-book is the one generally followed in later school life, and is just as easily taught to beginners as the above.

$$\begin{array}{r} 456 \\ 209 \\ \hline 4104 \\ 000 \\ 912 \\ \hline \end{array}$$

346. Where the multiplicands are small, as in nearly all of these examples, the product by the fraction should be determined "mentally" and written in its place. A pupil should not be encouraged to waste time by indicating on another part of his slate that 64 is to be divided by 8, and that this product is to be multiplied by 3, and doing all this work to reach a result that can be readily obtained without any writing whatever. In Nos. 78 and 88, such pupils as need to use the pencil in multiplying by the fraction should be permitted to do so. The teaching of the common method of multiplying by a mixed number is taken up at the beginning of the next chapter.

$$\begin{array}{r} 8)64 \\ 8 \\ \hline 3 \\ \hline 24 \end{array}$$

$$\begin{array}{r} 64 \\ 1\frac{3}{8} \\ \hline 24 \\ 64 \\ \hline 88 \end{array}$$

VIII

NOTES ON CHAPTER FIVE

347. The denominators of fractional multipliers have heretofore been factors of the multiplicands, and the latter have been, as a rule, small numbers. With the introduction of larger numbers and the occasional use of multiplicands that are not multiples of the denominators of the fractions in the multiplier, it becomes necessary to furnish pupils with a general method of dealing with this class of examples. (See Arithmetic, Art. 347.)

348. In multiplying 27 by $13\frac{1}{3}$, some pupils may be tempted to follow the rule, and to multiply 27 by the numerator 1. In the first few examples this may be permitted, but the scholars should soon be taught to discontinue the practice, and to divide the multiplicand without rewriting it. (See Art. 178.)

$$\begin{array}{r} 27 \\ 13\frac{1}{3} \\ \hline 3)\ 27 \\ \hline 9 \end{array}$$

etc.

350. In adding 56 and 17, the pupil should first combine 56 and 10 to make 66, and then add 7. (See Art. 286.)

351. Children taught subtraction by the "building-up" method will ascertain how many must be added to 19 to make 66, by saying 19 and 40 are 59, and 7 are 66; or 19 and 7 are 26, and 40 are 66. While the second plan is easier in some respects, it gives the 40 and the 7 of the result in the reverse order, which makes it necessary for the pupils to transpose them. In this respect, the first plan is more satisfactory.

When the other method of subtraction is practiced in slate work, 66 is first diminished by 10 and then by 9. To find the difference between 94 and 76, the pupil takes 70 from 94, leaving 24, and from this remainder takes 6.

352. In multiplying 24 by 4 the pupil begins at the tens. Four times 20 are 80, to which is added 4×4, making 96.

353. While nearly the whole class will learn to give answers mentally to the previous combinations, it may be necessary to use the division drills as "sight" work chiefly.

359. See Art. 319.

362. Oral problems involving several operations, or those of an unfamiliar type, should be solved from the book as "sight" work, and should be followed later on by similar questions answered without seeing the numbers. No. 5 is of the second kind; and it might be well to place it on the board, writing "2 thirds" and "1 third" to express the parts, instead of employing the fractional form or that given in the book. In No. 7, the quotient of $60 \div 40$ will be expressed by $1\frac{1}{2}$, instead of the $1\frac{20}{40}$ obtained by writing the remainder over the divisor. No. 5 should not be made an excuse for teaching a method of obtaining the cost of the whole when that of a part is given.

These examples are introduced to give variety to the work, to lay a foundation for subsequent systematic treatment of problems of this kind, and to give a pupil an opportunity to use his thinking powers. The way to deprive them of value is to "explain" how they should be done, or to require from the scholars too much analysis.

363. If the school does not own these measures, the teacher should endeavor to secure the loan of a quart, a peck, and a bushel, for a few hours, at least. Sawdust could be used to show pupils that the peck contains eight quarts, etc.

364. While many of the problems of this article resemble the previous oral problems, it may be advisable to solve a number of them as "sight" work, changing the numbers when necessary. The first may be read "How many 200-lb. barrels can be filled from 6,000 lb.?" In the second and third, the fractions may be omitted. The cost of the calico and of the ribbon in No. 4 may be made 10¢. Nos. 5 and 6 need no change, perhaps.

370–372. Do not waste time by endeavoring to use these examples to explain "carrying" or the local value of digits.

374. The answers should be written directly from the book. Do not permit scholars to copy the examples on their slates.

377. First, perform operations on the quantities enclosed within the parentheses.

384. Very little preliminary explanation will be needed. Place

(*a*)	?	(*a*) on the blackboard, and ask a	(*b*)	?
	125	pupil to write the missing number		125
	632	in its place, one figure at a time,		632
	999	beginning with the units' figure.		1000

Have another pupil work (*b*) in the same way. Nos. 1 to 5 may be used as a class exercise, each pupil writing only the answer on his paper, the examples being placed on the board.

385. In many German schools, children are not permitted in long division to write the partial products. Examples 6–23 are given to train pupils to omit these products when the quotient contains but one figure. After a few of them are worked on the board, the answers to the others may be written by all the pupils, as suggested in the preceding article. In writing the answers, the pupils should first set down the quotient figure, then the divisor as the denominator of a fraction, and lastly the remainder as a numerator. (See Art. 563, p. 55.)

386–388. These examples should be placed on the board, and the pupils should write the results one figure at a time.

397–401. See Art. 321.

405–406. See Arts. 306 and 307.

407. Prove the correctness of the grand total by comparing the total of the 6th column with that of the 11th row.

412. Permit the pupils to use their own method of working these examples, and avoid giving unnecessary assistance.

413–414. Example 1 should be omitted where pupils do not receive marks that are thus averaged. No. 2 may also be omitted if the word "average" is not understood by the pupils.

424–426. See Arts. 286–290, page 34.

429. In Examples 1, 2, 5, 9, etc., it will hardly be necessary to inform the pupils that 1 is not considered a factor of a number.

SUPPLEMENT

DEFINITIONS, PRINCIPLES, AND RULES

A **Unit** is a single thing.

A **Number** is a unit or a collection of units.

The **Unit of a Number** is one of that number.

Like Numbers are those that express units of the same kind.

Unlike Numbers are those that express units of different kinds.

A **Concrete Number** is one in which the unit is named.

An **Abstract Number** is one in which the unit is not named.

Notation is expressing numbers by characters.

Arabic Notation is expressing numbers by figures.

Roman Notation is expressing numbers by letters.

Numeration is reading numbers expressed by characters.

The **Place of a Figure** is its position in a number.

A figure standing alone, or in the first place at the right of other figures, expresses *ones*, or *units of the first order*.

A figure in the second place expresses *tens*, or *units of the second order*.

A figure in the third place expresses *hundreds*, or *units of the third order;* and so on.

A **Period** is a group of three orders of units, counting from right to left.

RULE FOR NOTATION. — *Begin at the left, and write the hundreds, tens, and units of each period in succession, filling vacant places and periods with ciphers.*

RULE FOR NUMERATION. — *Beginning at the right, separate the number into periods.*

Beginning at the left, read the numbers in each period, giving the name of each period except the last.

ADDITION

Addition is finding a number equal to two or more given numbers.

Addends are the numbers added.

The **Sum,** or **Amount,** is the number obtained by addition.

PRINCIPLE. — *Only like numbers, and units of the same order can be added.*

RULE. — *Write the numbers so that units of the same order shall be in the same column.*

Beginning at the right, add each column separately, and write the sum, if less than ten, under the column added.

When the sum of any column exceeds nine, write the units only, and add the ten or tens to the next column.

Write the entire sum of the last column.

SUBTRACTION

Subtraction is finding the difference between two numbers.

The **Subtrahend** is the number subtracted.

The **Minuend** is the number from which the subtrahend is taken.

The **Remainder,** or **Difference,** is the number left after subtracting one number from another.

PRINCIPLES. — *Only like numbers and units of the same order can be subtracted.*

The sum of the difference and the subtrahend must equal the minuend.

RULES. — I. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

Beginning at the right, find the number that must be added to the first figure of the subtrahend to produce the figure in the corresponding order of the minuend, and write it below. Proceed in this way until the difference is found.

If any figure in the subtrahend is greater than the corresponding figure in the minuend, find the number that must be added to the former to produce the latter increased by ten; then add one to the next order of the subtrahend and proceed as before.

II. *Beginning at the units' column, subtract each figure of the subtrahend from the corresponding figure of the minuend and write the remainder below.*

If any figure of the subtrahend is greater than the corresponding figure in the minuend, add ten to the latter and subtract; then, (a) *add one to the next order of the subtrahend and proceed as before; or,* (b) *subtract one from the next order of the minuend and proceed as before.*

MULTIPLICATION

Multiplication is taking one number as many times as there are units in another number.

The **Multiplicand** is the number taken or multiplied.

The **Multiplier** is the number that shows how many times the multiplicand is taken.

The **Product** is the result obtained by multiplication.

PRINCIPLES. — *The multiplier must be an abstract number.*

The multiplicand and the product are like numbers.

The product is the same in whatever order the numbers are multiplied.

RULE. — *Write the multiplier under the multiplicand, placing units of the same order in the same column.*

Beginning at the right, multiply the multiplicand by the number of units in each order of the multiplier in succession. Write the

figure of the lowest order in each partial product under the figure of the multiplier that produces it. Add the partial products.

To multiply by 10, 100, 1000, etc.

RULE. — *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

DIVISION

Division is finding how many times one number is contained in another, or finding one of the equal parts of a number.

The **Dividend** is the number divided.

The **Divisor** is the number contained in the dividend.

The **Quotient** is the result obtained by division.

PRINCIPLES. — *When the divisor and the dividend are like numbers, the quotient is an abstract number.*

When the divisor is an abstract number, the dividend and the quotient are like numbers.

The product of the divisor and the quotient, plus the remainder, if any, is equal to the dividend.

RULE. — *Write the divisor at the left of the dividend with a line between them.*

Find how many times the divisor is contained in the fewest figures on the left of the dividend, and write the result over the last figure of the partial dividend. Multiply the divisor by this quotient figure, and write the product under the figures divided. Subtract the product from the partial dividend used, and to the remainder annex the next figure of the dividend for a new dividend.

Divide as before until all the figures of the dividend have been used.

If any partial dividend will not contain the divisor, write a cipher in the quotient, and annex the next figure of the dividend.

If there is a remainder after the last division, write it after the quotient with the divisor underneath.

FACTORING

An **Exact Divisor** of a number is a number that will divide it without a remainder.

An **Odd Number** is one that cannot be exactly divided by two.

An **Even Number** is one that can be exactly divided by two.

The **Factors** of a number are the numbers that multiplied together produce that number.

A **Prime Number** is a number that has no factors.

A **Composite Number** is a number that has factors.

A **Prime Factor** is a prime number used as a factor.

A **Composite Factor** is a composite number used as a factor.

Factoring is separating a number into its factors.

To find the Prime Factors of a Number.

RULE. — *Divide the number by any prime factor. Divide the quotient, if composite, in like manner; and so continue until a prime quotient is found. The several divisors and the last quotient will be the prime factors.*

CANCELLATION

Cancellation is rejecting equal factors from dividend and divisor.

PRINCIPLE. — *Dividing dividend and divisor by the same number does not affect the quotient.*

GREATEST COMMON DIVISOR

A **Common Factor** (divisor or measure) is a number that is a factor of each of two or more numbers.

A **Common Prime Factor** is a prime number that is a factor of each of two or more numbers.

The **Greatest Common Factor** (divisor or measure) is the largest number that is a factor of each of two or more numbers.

Numbers are prime to each other when they have no common factor.

The greatest common divisor of two or more numbers is the product of their common prime factors.

PRINCIPLES. — *A common divisor of two numbers is a divisor of their sum, and also of their difference.*

A divisor of a number is a divisor of every multiple of that number; and a common divisor of two or more numbers is a divisor of any of their multiples.

To find the Common Prime Factors of Two or More Numbers.

RULE. — *Divide the numbers by any common prime factors, and the quotients in like manner, until they have no common factor; the several divisors are the common prime factors.*

To find the Greatest Common Divisor of Numbers that are Easily Factored.

RULE. — *Separate the numbers into their prime factors; the product of those that are common is the greatest common divisor.*

To find the Greatest Common Divisor of Numbers that are not Easily Factored.

RULE. — *Divide the greater number by the less; then divide the last divisor by the last remainder, continuing until there is no remainder. The last divisor is the greatest common divisor.*

If there are more than two numbers, find the greatest common divisor of two of them; then of that divisor and another of the numbers until all of the numbers have been used. The last divisor is the greatest common divisor.

LEAST COMMON MULTIPLE

A **Multiple** of a number is a number that exactly contains that number.

A **Common Multiple** of two or more numbers is a number that is a multiple of each of them.

The **Least Common Multiple** of two or more numbers is the smallest number that is a common multiple of them.

PRINCIPLES. — *A multiple of a number contains all the prime factors of that number.*

A common multiple of two or more numbers contains each of the prime factors of those numbers.

The Least Common Multiple of two or more numbers contains only the prime factors of each of the numbers.

To find the Least Common Multiple of Two or More Numbers.

RULE. — *Divide by any prime number that is an exact divisor of two or more of the numbers, and write the quotients and undivided numbers below. Divide these numbers in like manner, continuing until no two of the remaining numbers have a common factor. The product of the divisors and remaining numbers is the least common multiple.*

FRACTIONS

A **Fraction** is one or more of the equal parts of anything.

The **Unit of a Fraction** is the number or thing that is divided into equal parts.

A **Fractional Unit** is one of the equal parts into which the number or thing is divided.

The **Terms of a Fraction** are its numerator and its denominator.

The **Denominator** of a fraction shows into how many parts the unit is divided.

The **Numerator** of a fraction shows how many of the parts are taken.

A fraction indicates division; the numerator being the dividend and the denominator the divisor.

The **Value of a Fraction** is the quotient of the numerator divided by the denominator.

Fractions are divided into two classes — **Common** and **Decimal.**

A **Common Fraction** is one in which the unit is divided into any number of equal parts.

A common fraction is expressed by writing the numerator above the denominator with a dividing line between.

Common fractions consist of three principal classes — **Simple, Compound,** and **Complex.**

A **Simple Fraction** is one whose terms are whole numbers.

A **Proper Fraction** is a simple fraction whose numerator is less than its denominator.

An **Improper Fraction** is a simple fraction whose numerator equals or exceeds its denominator.

A **Compound Fraction** is a fraction of a fraction.

A **Complex Fraction** is one having a fraction in its numerator, or in its denominator, or in both.

A **Mixed Number** is a whole number and a fraction written together.

The **Reciprocal of a Number** is one divided by that number.

The **Reciprocal of a Fraction** is one divided by the fraction, or the fraction inverted.

PRINCIPLES. — *Multiplying the numerator or dividing the denominator multiplies the fraction.*

Dividing the numerator or multiplying the denominator divides the fraction.

Multiplying or dividing both terms of a fraction by the same number does not alter the value of the fraction.

Reduction of fractions is changing their terms without altering their value.

To reduce a Fraction to Higher Terms.

RULE. — *Multiply both numerator and denominator by the same number.*

To reduce a Fraction to its Lowest Terms.

RULE. — *Divide both terms of the fraction by their greatest common divisor.*

A fraction is in its lowest terms when the numerator and the denominator are prime to each other.

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To reduce a Mixed Number to an Improper Fraction.

RULE. — *Multiply the whole number by the denominator; to the product add the numerator; and place the sum over the denominator.*

To reduce an Improper Fraction to a Whole or to a Mixed Number.

RULE. — *Divide the numerator by the denominator.*

A **Common Denominator** is a denominator common to two or more fractions.

The **Least Common Denominator** is the smallest denominator common to two or more fractions.

To reduce Fractions to their Least Common Denominator.

RULE. — *Find the least common multiple of all the denominators for the least common denominator. Divide this multiple by the denominator of each fraction, and multiply the numerator by the quotient.*

ADDITION OF FRACTIONS

PRINCIPLE. — *Only like fractions can be added.*

RULE. — *Reduce the fractions, if necessary, to a common denominator, and over it write the sum of the numerators.*

If there are mixed numbers, add the fractions and the whole numbers separately, and unite the results.

SUBTRACTION OF FRACTIONS

PRINCIPLE. — *Only like fractions can be subtracted.*

RULE. — *Reduce the fractions, if necessary, to a common denominator, and over it write the difference between the numerators.*

If there are mixed numbers subtract the fractions and the whole numbers separately, and unite the results.

MULTIPLICATION OF FRACTIONS

RULE. — *Reduce whole and mixed numbers to improper fractions; cancel the factors common to numerators and denominators, and write the product of the remaining factors in the numerators over the product of the remaining factors in the denominators.*

DIVISION OF FRACTIONS

RULES. — I. *Reduce whole and mixed numbers to improper fractions. Reduce the fractions to a common denominator. Divide the numerator of the dividend by the numerator of the divisor.*

II. *Invert the divisor and proceed as in multiplication of fractions.*

To reduce a Complex Fraction to a Simple One.

RULES. — I. *Multiply the numerator of the complex fraction by its denominator inverted.*

II. *Multiply both terms by the least common multiple of the denominators.*

DECIMALS

A **Decimal Fraction** is one in which the unit is divided into tenths, hundredths, thousandths, etc.

A **Decimal** is a decimal fraction whose denomination is indicated by the number of places at the right of the decimal point.

The **Decimal Point** is the mark used to locate units.

A **Mixed Decimal** is a whole number and a decimal written together.

A **Complex Decimal** is a decimal with a common fraction written at its right.

To write Decimals.

RULE. — *Write the numerator; and from the right, point off as many decimal places as there are ciphers in the denominator, prefixing ciphers, if necessary, to make the required number.*

To read Decimals.

RULE. — *Read the numerator, and give the name of the right-hand order.*

PRINCIPLES. — *Prefixing ciphers to a decimal diminishes its value.*

Removing ciphers from the left of a decimal increases its value. Annexing ciphers to a decimal or removing ciphers from its right does not alter its value.

To reduce a Decimal to a Common Fraction.

RULE.— *Write the figures of the decimal for the numerator, and* 1, *with as many ciphers as there are places in the decimal, for the denominator, and reduce the fraction to its lowest terms.*

To reduce a Common Fraction to a Decimal.

RULE. — *Annex decimal ciphers to the numerator, and divide it by the denominator.*

To reduce Decimals to a Common Denominator.

RULE. — *Make their decimal places equal by annexing ciphers.*

ADDITION AND SUBTRACTION OF DECIMALS

Decimals are added and subtracted the same as whole numbers.

MULTIPLICATION OF DECIMALS

RULE. — *Multiply as in whole numbers, and from the right of the product, point off as many decimal places as there are decimal places in both factors.*

DIVISION OF DECIMALS

RULE. — *Make the divisor a whole number by removing the decimal point, and make a corresponding change in the dividend. Divide as in whole numbers, and place the decimal point in the quotient under (or over) the new decimal point in the dividend.*

ACCOUNTS AND BILLS

A Debtor is a person who owes another.
A **Creditor** is a person to whom a debt is due.

An **Account** is a record of debits and credits between persons doing business.

The **Balance** of an account is the difference between the debit and credit sides.

A **Bill** is a written statement of an account.

An **Invoice** is a written statement of items, sent with merchandise.

A **Receipt** is a written acknowledgment of the payment of part or all of a debt.

A bill is receipted when the words, "Received Payment," are written at the bottom, signed by the creditor, or by some person duly authorized.

DENOMINATE NUMBERS

A **Measure** is a standard established by law or custom, by which distance, capacity, surface, time, or weight is determined.

A **Denominate Unit** is a unit of measure.

A **Denominate Number** is a denominate unit or a collection of denominate units.

A **Simple Denominate Number** consists of denominate units of one kind.

A **Compound Denominate Number** consists of denominate units of two or more kinds.

A **Denominate Fraction** is a fraction of a denominate number.

A denominate fraction may be either **common** or **decimal.**

Reduction of denominate numbers is changing them from one denomination to another without altering their value.

Reduction Descending is changing a denominate number to one of a lower denomination.

RULE. — *Multiply the highest denomination by the number required to reduce it to the next lower denomination, and to the product add the units of that lower denomination, if any. Proceed in this manner until the required denomination is reached.*

Reduction Ascending is changing a denominate number to one of a higher denomination.

RULE. — *Divide the given denomination successively by the numbers that will reduce it to the required denomination. To this quotient annex the several remainders.*

To find the Time between Dates.

RULE. — *When the time is less than one year, find the exact number of days; if greater than one year, find the time by compound subtraction, taking* 30 *days to the month.*

PERCENTAGE

Per Cent means hundredths.

Percentage is computing by hundredths.

The elements involved in percentage are the **Base, Rate, Percentage, Amount,** and **Difference.**

The **Base** is the number of which a number of hundredths is taken.

The **Rate** indicates the number of hundredths to be taken.

The **Percentage** is one or more hundredths of the base.

The **Amount** is the base increased by the percentage.

The **Difference** is the base diminished by the percentage.

To find the Percentage when the Base and Rate are Given.

RULE. — *Multiply the base by the rate expressed as hundredths.*

To find the Rate when the Percentage and Base are Given.

RULE. — *Divide the percentage by the base.*

To find the Base when the Percentage and Rate are Given.

RULE. — *Divide the percentage by the rate expressed as hundredths.*

To find the Base when the Amount and Rate are Given.

RULE. — *Divide the amount by* $1 +$ *the rate expressed as hundredths.*

To find the Base when the Difference and Rate are Given.

RULE. — *Divide the difference by* 1 — *the* ***rate expressed as hundredths.***

PROFIT AND LOSS

Profit or **Loss** is the difference between the buying and selling prices.

In Profit and Loss,

The buying price, or cost, is the ***base.***

The rate per cent profit or loss is the *rate.*

The profit or loss is the ***percentage.***

The selling price is the *amount* or *difference*, according as it is more or less than the buying price.

COMMERCIAL DISCOUNT

Commercial Discount is a percentage deducted from the list price of goods, the face of a bill, etc.

The **Net Price** of goods is the sum received for them.

In Commercial Discount,

The list price, or
The face of the bill } is the ***base.***

The rate per cent discount is the ***rate.***

The discount is the *percentage.*

The list price diminished by the discount is the *difference.*

In successive discounts, the first discount is made from the list price or the face of the bill; the second discount, from the list price or face of the bill diminished by the first discount; and so on.

COMMISSION

Commission is a percentage allowed an agent for his services.

A Commission Agent is one who transacts business on commission.

A **Consignment** is the merchandise forwarded to a commission agent.

The **Consignor** is the person who sends the merchandise.

The **Consignee** is the person to whom the merchandise is sent.

The **Net Proceeds** is the sum remaining after all charges have been deducted.

In buying, the commission is a percentage of the *buying price;* in selling, a percentage of the *selling price;* in collecting, a percentage of the *sum collected;* hence:

The sum invested, or the sum collected } is the *base.*

The rate per cent commission is the *rate.*

The commission is the *percentage.*

The sum invested increased by the commission is the *amount.*

The sum collected diminished by the commission is the *difference.*

INSURANCE

Insurance is a contract of indemnity.

Insurance is of three kinds — **Fire, Marine,** and **Life.**

Fire Insurance is indemnity against loss of property by fire.

Marine Insurance is indemnity against loss of property by the casualities of navigation.

Life Insurance is indemnity against loss of life.

The **Insurance Policy** is the contract setting forth the liability of the insurer.

The **Policy Face** is the amount of insurance.

The **Premium** is the price paid for insurance.

The **Insurer,** or **Underwriter,** is the company issuing the policy.

The **Insured** is the person for whose benefit the policy is issued.

In Insurance,

The policy face is the *base.*

The rate per cent premium is the *rate.*

The premium is the *percentage.*

TAXES

A **Tax** is a sum of money levied on persons or property for public purposes.

A **Personal**, or **Poll Tax**, is a tax on the person.

A **Property Tax** is a tax of a certain per cent on the assessed value of property.

Property may be either personal or real.

Personal Property consists of such things as are movable.

Real Property is that which is fixed, or immovable.

In Taxes,

The assessed value is the *base.*

The rate of taxation is the *rate.*

The tax is the *percentage.*

DUTIES

Duties are taxes on imported goods.

Duties are either **Specific** or **Ad Valorem.**

A **Specific Duty** is a tax on goods without regard to cost.

An **Ad Valorem** duty is a tax of a certain per cent on the cost of goods.

In Ad Valorem Duties,

The cost of the goods is the *base.*

The rate per cent duty is the *rate.*

The ad valorem duty is the *percentage.*

INTEREST

Interest is the sum paid for the use of money.

The **Principal** is the sum loaned.

The **Amount** is the sum of the principal and interest.

The **Rate of Interest** is the rate per cent for one year.

The **Legal Rate** is the rate fixed by law.

Usury is interest at a higher rate than that fixed by law.

Simple Interest is interest on the principal only.

To find the Interest when the Principal, Time, and Rate are Given.

RULE. — *Multiply the principal by the rate expressed as hundredths, and this product by the time expressed in years.*

To find the Time when the Principal, Interest, and Rate are Given.

RULE. — *Divide the given interest by the interest for one year.*

To find the Rate when the Principal, Interest, and Time are Given.

RULE. — *Divide the given interest by the interest at one per cent.*

To find the Principal when the Interest, Rate, and Time are Given.

RULE. — *Divide the given interest by the interest on* $1.

To find the Principal when the Amount and Time and Rate are Given.

RULE. — *Divide the given amount by the amount of* $1.

INTEREST BY ALIQUOT PARTS.

To find the Interest for Years, Months, and Days.

RULE. — *Find the interest for one year and take this as many times as there are years.*

Take the greatest number of the given months that equals an aliquot part of a year and find the interest for this time. Take aliquot parts of this for the remaining months.

In the same manner find the interest for the days.

The sum of these interests will be the interest required.

To find the Interest when the Time is Less than a Year.

RULE. — *Find the interest for the time in months or days that will gain one per cent of the principal.*

Find by aliquot parts, as in the first rule, the interest for the remaining time.

The sum of these interests will be the interest required.

INTEREST BY SIX PER CENT METHOD.

To find the Interest at 6%.

RULE. — **For Years:** *Multiply the principal by the rate expressed as hundredths, and that product by the number of years.*

For Months: *Move the decimal point two places to the left, and multiply by one-half the number of months.*

For Days: *Move the decimal point three places to the left, and multiply by one-sixth the number of days.*

To find the interest at any other rate per cent, divide the interest at 6% by 6, and multiply the quotient by the given rate.

To find Exact Interest.

RULE. — *Multiply the principal by the rate expressed as hundredths, and that product by the time expressed in years of* 365 *days.*

ANNUAL INTEREST

Annual Interest is interest payable annually. If not paid when due, annual interest draws simple interest.

To find the Amount Due on a Note with Annual Interest, when the Interest has not been Paid Annually.

RULE. — *Find the interest on the principal for the entire time, and on each annual interest for the time it remained unpaid. The sum of the principal and all the interest is the amount due.*

COMPOUND INTEREST

Compound Interest is interest on the principal and on the unpaid interest, which is added to the principal at regular intervals. The interest may be compounded annually, semi-annually, quarterly, etc., according to agreement.

To find Compound Interest.

RULE. — *Find the amount of the given principal for the first period. Considering this as a new principal, find the amount of*

it for the next period, continuing in this manner for the given time.

Find the difference between the last amount and the given principal, which will be the compound interest.

PARTIAL PAYMENTS

Partial Payments are part payments of a note or debt. Each payment is recorded on the back of the note or the written obligation.

UNITED STATES RULE. — *Find the amount of the principal to the time when the payment or the sum of two or more payments equals or exceeds the interest.*

From this amount deduct the payment or sum of payments.

Use the balance then due as a new principal, and proceed as before.

MERCHANTS' RULE. — *Find the amount of an interest-bearing note at the time of settlement.*

Find the amount of each credit from its time of payment to the time of settlement; subtract their sum from the amount of the principal.

BANK DISCOUNT

Bank Discount is a percentage retained by a bank for advancing money on a note before it is due.

The **Sum Discounted** is the face of the note, or if interest-bearing, the amount of the note at maturity.

The **Term of Discount** is the number of days from the day of discount to the day of maturity.

The **Bank Discount** is the interest on the sum discounted for the term of discount.

The **Proceeds** of a note is the sum discounted less the bank discount.

Problems in bank discount are calculated as problems in interest.

In Bank Discount,

The sum discounted is the *principal.*

The rate of discount is the *rate of interest.*

The term of discount is the *time.*

The bank discount is the *proceeds.*

EXCHANGE

Exchange is making payments at a distance by means of drafts or bills of exchange.

Domestic Exchange is exchange between places in the same country.

Foreign Exchange is exchange between different countries.

Exchange is *at par* when a draft, or bill, sells for its face value; *at a premium* when it sells for more than its face value; *at a discount* when it sells for less.

The cost of a sight draft is the face of the draft increased by the premium, or diminished by the discount.

The cost of a time draft is the face of the draft increased by the premium, or diminished by the discount, and this result diminished by the bank discount.

To find the Cost of a Draft.

RULE. — *Find the cost of* $1 *of the draft; multiply this by the face of the draft.*

To find the Face of a Draft.

RULE. — *Divide the cost of the draft by the cost of* $1 *of the draft.*

EQUATION OF PAYMENTS

Equation of Payments is a method of ascertaining at what time several debts due at different times may be settled by a single payment.

The **Equated Time** of payment is the time when the several debts may be equitably settled by one payment.

The **Term of Credit** is the time the debt has to run before it becomes due.

The Average Term of Credit is the time the debts due at different times have to run, before they may be equitably settled by one payment.

To find the Equated Time of Payment when the Terms of Credit begin at the Same Date.

RULE. — *Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit.*

Add the average term of credit to the date of the debts, and the result will be the equated time of payment.

To find the Equated Time when the Terms of Credit begin at Different Dates.

RULE. — *Find the date at which each debt becomes due. Select the earliest date as a standard.*

Multiply each debt by the number of days between the standard date and the date when the debt becomes due, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit from the standard date.

Add the average term of credit to the standard date, and the result will be the equated time of payment.

RATIO

Ratio is the relation one number bears to another of the same kind.

The **Terms** of the ratio are the numbers compared.

The **Antecedent** is the first term.

The **Consequent** is the second term.

The antecedent and consequent form a *couplet*.

PRINCIPLES. — See Fractions.

PROPORTION

A **Proportion** is formed by two equal ratios.

The **Extremes** of a proportion are the first and last terms.

The **Means** of a proportion are the second and third terms.

PRINCIPLES. — *The product of the means is equal to the product of the extremes.*

Either mean equals the product of the extremes divided by the other mean.

Either extreme equals the product of the means divided by the other extreme.

RULE FOR PROPORTION. — *Represent the required term by x.*

Arrange the terms so that the required term and the similar known term may form one couplet, the remaining terms the other.

If the required term is in the extremes, divide the product of the means by the given extreme.

If the required term is in the means, divide the product of the extremes by the given mean.

PARTNERSHIP

Partnership is an association of two or more persons for business purposes.

The **Partners** are the persons associated.

The **Capital** is that which is invested in the business.

The **Assets** are the partnership property.

The **Liabilities** are the partnership debts.

To find the Profit, or Loss, of Each Partner when the Capital of Each is Employed for the Same Period of Time.

RULE. — *Find the part of the entire profit, or loss, that each partner's capital is of the entire capital.*

To find the Profit, or Loss, of Each Partner when the Capital of Each is Employed for Different Periods of Time.

RULE. — *Find each partner's capital for one month, by multiplying the amount he invests by the number of months it is employed; then find the part of the entire profit, or loss, that each partner's capital for one month is of the entire capital for one month.*

INVOLUTION

A **Power** of a number is the product obtained by using that number a certain number of times as a factor.

The **First Power** of a number is the number itself.

The **Second Power** of a number, or the **Square**, is the product of a number taken twice as a factor.

The **Third Power** of a number, or the **Cube**, is the product of a number taken three times as a factor.

An **Exponent** is a small figure written a little to the right of the upper part of a number to indicate the power.

Involution is finding any power of a number.

To find the Power of a Number.

RULE. — *Take the number as a factor as many times as there are units in the exponent.*

EVOLUTION

A **Root** is one of the equal factors of a number.

The **Square Root** of a number is one of its two equal factors.

The **Cube Root** of a number is one of its three equal factors.

Evolution is finding any root of a number.

Evolution may be indicated in two ways: by the *Radical Sign*, $\sqrt{\ }$, or by a *fractional exponent.*

The **Index** of a root is a small figure placed a little to the left of the upper part of the radical sign, to indicate what root is to be found. In expressing square root, the index is omitted.

In the fractional exponent, the numerator indicates the power to which the number is to be raised; the denominator indicates the root to be taken of the number thus raised.

To find the Square Root of a Number.

RULE. — *Point off in periods of two figures, commencing at units. Find the greatest square in the first period and place the root in the quotient. Subtract this square from the first period, and bring down the next period.*

Multiply the quotient figure by two, and use it as a trial divisor. Place the second figure in the quotient, and annex it also to the trial divisor. Then multiply the figures in the trial divisor by the second quotient figure, and subtract.

Bring down the next period, and proceed as before until the square root is found.

To find the Square Root of a Fraction.

RULE. — *Reduce the fraction to its simplest form, and find the square root of each term separately.*

To find the Cube Root of a Number.

RULE. — *Point off in periods of three figures each, beginning at units.*

Find the greatest cube in the first period and place the root in the quotient. Subtract this cube from the first period, and bring down the next period.

Multiply the square of the first quotient figure by three and annex two ciphers for a trial divisor. Place the second figure in the quotient. Then, to the trial divisor add three times the product of the first and second figures, also the square of the second. Multiply this sum by the second figure and subtract.

Bring down the next period, and proceed as before until the cube root is found.

To find the Cube Root of a Fraction.

RULE. — *Reduce the fraction to its simplest form, and find the cube root of each term separately.*

STOCKS AND BONDS.

Capital Stock is the money or property employed by a corporation in its business.

A **Share** is one of the equal divisions of capital stock.

The **Stockholders** are the owners of the capital stock.

The **Par Value** of stock is the face value.

The **Market Value** of stock is the sum for which it may be sold.

Stock is at a *premium* when the market value is above the par value; at a *discount*, when below par.

Bonds are interest-bearing notes issued by a government or a corporation.

A **Dividend** is a percentage apportioned among the stockholders.

A **Stock Broker** is a person who deals in stocks.

Brokerage is a percentage allowed a stock broker for his services.

In Stocks and Bonds,

The par value is the *base.*

The rate per cent premium, or discount, is the *rate.*

The premium, discount, or dividend } is the *percentage.*

The market value is the { *amount*, or *difference.*

NOTES, DRAFTS, AND CHECKS.

A **Promissory Note** is a written promise to pay a specified sum on demand, or at a specified time.

The **Face** of a note is the sum named in the note.

The **Maker** is the person who signs it.

The **Payee** is the person to whom the sum specified is to be paid.

The **Indorser** is the person who signs his name on the back of the note, thus becoming liable for its payment in case of default of the maker.

An **Interest-bearing Note** is one payable with interest.

If the words "with interest" are omitted, interest cannot be collected until after maturity.

A **Demand Note** is one payable when demand of payment is made.

A **Time Note** is one payable at a specified time.

A **Joint Note** is one signed by two or more persons who jointly promise to pay.

A **Joint and Several Note** is one signed by two or more persons who jointly and severally promise to pay.

In a joint note, each person is liable for the whole amount, but they must all be sued together. In the joint and several note, each is liable for the whole amount, and may be sued separately.

A **Negotiable Note** is one that may be transferred or sold. It contains the words " or bearer," or " or order."

A **Non-negotiable Note** is one not payable to the bearer, nor to the payee's order.

The **Maturity** of a note is the day on which it legally falls due.

A **Draft**, or **Bill of Exchange**, is a written order directing the payment of a specified sum of money.

The **Face** of a draft is the sum named in it.

The **Drawer** is the person who signs the draft.

The **Drawee** is the person ordered to pay the sum specified.

The **Payee** is the person to whom the sum specified is to be paid.

A **Sight Draft** is one payable when presented.

A **Time Draft** is one payable at a specified time.

An **Acceptance** of a time draft is an agreement by the drawee to pay the draft at maturity, which he signifies by writing across the face of the draft the word " accepted " with the date and his name.

A **Check** is an order on a bank or banker to pay a specified sum of money.

PRIMARY ARITHMETIC

ANSWERS.

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19. $418.75.
20. $207.41.
21. $9.90.
22. $21.50.
23. $38.35.
24. $40.13.
25. $18.59.
26. $112.75.

Page 101.

1. 378.
2. 210.
3. 496.
4. 609.
5. 744.
6. 1088.
7. 1944.
8. 2175.
9. 3139.
10. 1200.
11. 4104.
12. 944.
13. 819.
14. 2583.
15. 775.
16. 2180.
17. 3402.
18. 5376.
19. 5238.
20. 1342.

Page 108.

1. 50.
2. 68¾.
3. 96¾.
4. 72.
5. 100.
6. 96¼.
7. 85¼.
8. 70¼.
9. 86¼.
10. 43¼.
11. 78.

Page 110.

1. 143.
156.
2. 154.
168.
3. 165.
180.
4. 176.
192.
5. 187.
204.
6. 231.
252.

7. 242.
264.
8. 253.
276.
9. 264.
288.
10. 275.
300.
11. 341.
372.
12. 352.
384.
13. 363.
396.
14. 374.
408.
15. 451.
492.
16. 462.
504.
17. 473.
516.
18. 550.
600.
19. 561.
612.
20. 572.
624.
21. 671.
732.
22. 682.
744.
23. 803.
876.
24. 924.
1008.
25. 1023.
1116.
26. 1221.
1332.
27. 2343.
2556.
28. 3564.
3888.
29. 5016.
5472.
30. 5577.
6084.
31. 6765.
7380.
32. 7920.
8640.
33. 8976.
9792.
34. 10,164.
11,088.
35. 11,682.
12,744.
36. 25,795.
28,140.
37. 34,364.
37,488.
38. 49,676.
54,192.
39. 63,019.
68,748.
40. 70,224.
76,608.
41. 86,988.
94,896.
42. 90,970.
99,240.
43. 88,935.
97,020.
44. 86,768.
94,656.
45. 82,137.
89,604.
46. 69,520.
75,840.
47. 64,988.
70,896.
48. 50,237.
54,804.
49. 34,045.
37,140.
50. 32,989.
35,988.

Page 111.

51. 12.
11.
52. 24.
22.
53. 36.
33.
54. 48.
44.
55. 60.
55.
56. 72.
66.
57. 84.
77.
58. 96.
88.
59. 108.
99.
60. 120.
110.
61. 144.
132.
62. 168.
154.
63. 252.
231.
64. 360.
330.
65. 492.
451.
66. 672.
616.
67. 828.
759.
68. $170\frac{6}{11}$.
$156\frac{4}{12}$.
69. $213\frac{2}{11}$.
$195\frac{5}{12}$.
70. $333\frac{7}{11}$.
$305\frac{10}{12}$.
71. $389\frac{3}{11}$.
$356\frac{9}{12}$.
72. 482.
$441\frac{10}{12}$.
73. $613\frac{6}{11}$.
$562\frac{8}{12}$.
74. $712\frac{8}{11}$.
$652\frac{11}{12}$.
75. $811\frac{5}{11}$.
$743\frac{10}{12}$.
76. $857\frac{4}{11}$.
$785\frac{11}{12}$.
77. 936.
858.
78. $1110\frac{10}{11}$.
$1018\frac{4}{12}$.
79. 1284.
1177.
80. 1560.
1430.
81. 3300.
3025.
82. 4272.
3916.
83. 7116.
6523.
84. 8484.
7777.
85. $981\frac{5}{11}$.
$899\frac{8}{12}$.

86. $1109\frac{10}{11}$.
$1017\frac{5}{12}$.
87. $1362\frac{6}{11}$.
1249.
88. $1518\frac{2}{11}$.
$1391\frac{8}{12}$.
89. $1637\frac{1}{11}$.
$1500\frac{8}{12}$.
90. $1910\frac{2}{11}$.
1751.
91. $2243\frac{7}{11}$.
$2056\frac{8}{12}$.
92. $2520\frac{7}{11}$.
$2310\frac{7}{12}$.
93. $2782\frac{8}{11}$.
$2550\frac{5}{12}$.
94. $3301\frac{7}{11}$.
$3026\frac{6}{12}$.
95. $4545\frac{4}{11}$.
$4166\frac{7}{12}$.
96. $4893\frac{4}{11}$.
$4485\frac{7}{12}$.
97. 5726.
$5248\frac{10}{12}$.
98. $6391\frac{3}{11}$.
$5858\frac{8}{12}$.
99. 54.
100. 41.
101. 67.
102. 81.
103. 75.
104. 72.
105. $4\frac{1}{4}$.
106. 8.
107. 7.
108. 11.
109. 124.
110. 124.
111. 243.
112. 129.
113. 420.
114. 810.
115. 183.
116. 412.
117. 504.
118. 336.

Page 112.

119. 405.
120. 255.
121. 1089.
122. 1056.
123. 30.
124. 50.
125. 70.
126. 90.
127. 25.
128. 35.
129. 40.
130. 50.
131. 80.
132. 55.
133. 28.
134. 52.
135. 76.
136. 55.
137. 115.
138. 610.
139. $25\frac{1}{4}$.
140. $\frac{3}{4}$.
141. $1\frac{1}{4}$.
142. $32\frac{1}{4}$.
143. $49\frac{1}{4}$.
144. $19\frac{1}{8}$.
145. $58\frac{1}{2}$.
146. $19\frac{1}{2}$.
147. $24\frac{3}{4}$.
148. $52\frac{1}{4}$.

Page 115.

1. 450.
2. 640.
3. 850.
4. 1080.
5. 1330.
6. 1600.
7. 1890.
9. 2530.
10. 2880.
11. 2460.
12. 7020.
13. 13,800.
14. 22,800.
15. 34,020.
16. 47,460.
17. 63,120.
18. 80,100.
20. 96,360.
21. 91,800.
22. 99,240.
23. 99,150.
24. 99,040.
35. 15.
36. 15.
37. 14.
38. 16.
39. 26.
40. 58.
41. 143.
42. 236.
43. 469.
44. 616.
45. 572.
46. 906.
47. 1307.
48. 1923.

Page 116.

58. $36\frac{9}{20}$.
59. $28\frac{3}{30}$.
60. $132\frac{1}{40}$.
61. $127\frac{9}{50}$.
62. $126\frac{5}{60}$.
63. $129\frac{4}{70}$.
64. $142\frac{7}{80}$.
65. $149\frac{5}{90}$.
67. $184\frac{1}{110}$.
68. $199\frac{7}{120}$.
69. $356\frac{9}{110}$.
70. $534\frac{3}{90}$.
71. $786\frac{3}{80}$.
72. $1142\frac{1}{70}$.
89. $54\frac{11}{12}$.
90. $125\frac{14}{15}$.
91. $168\frac{8}{18}$.
92. $469\frac{18}{20}$.
93. $866\frac{12}{16}$.
94. $1541\frac{18}{30}$.
95. $2640\frac{14}{20}$.
96. $2422\frac{21}{30}$.
97. $2370\frac{15}{40}$.
98. $336\frac{7}{50}$.
99. $555\frac{18}{60}$.
100. $21\frac{8}{70}$.
101. $287\frac{45}{80}$.
102. $109\frac{88}{90}$.
103. $770\frac{88}{90}$.
104. $905\frac{88}{80}$.
105. $617\frac{20}{70}$.
106. $571\frac{20}{80}$.
107. $1084\frac{10}{30}$.
108. 82.
109. 144.
110. $339\frac{10}{20}$.
111. $59\frac{94}{110}$.
112. $60\frac{30}{120}$.
113. $156\frac{75}{110}$.
114. $155\frac{95}{120}$.
115. $1036\frac{8}{90}$.
116. $1014\frac{88}{90}$.
117. $176\frac{15}{70}$.

Page 117.

1. $34\frac{1}{2}$.

2. $48\frac{1}{2}$.
3. $75\frac{1}{2}$.
4. $88\frac{1}{2}$.
5. 83.
6. $42\frac{1}{2}$.
7. $62\frac{1}{2}$.
8. $26\frac{1}{2}$.
9. $83\frac{1}{2}$.
10. 78.
11. $53\frac{1}{2}$.
12. $84\frac{1}{2}$.
13. $83\frac{1}{2}$.
14. $93\frac{1}{2}$.
15. $90\frac{1}{2}$.
16. 96.
17. $94\frac{1}{2}$.
18. $99\frac{1}{2}$.
19. 81.
20. 96.
21. $3\frac{1}{2}$.
22. $17\frac{1}{2}$.
23. 5.
24. $37\frac{1}{2}$.
25. $42\frac{1}{2}$.
26. $52\frac{1}{2}$.
27. $7\frac{1}{2}$.
28. $58\frac{1}{2}$.
29. $7\frac{1}{2}$.
30. $43\frac{1}{2}$.
31. $46\frac{1}{2}$.
32. 13.
34. 15.
35. $36\frac{1}{2}$.
36. $19\frac{1}{2}$.
37. $37\frac{1}{2}$.
38. $13\frac{1}{2}$.
39. $7\frac{1}{2}$.
40. $34\frac{1}{2}$.

Page 119.

1. 182.
2. 299.
3. 793.
4. 715.
5. 196.
6. 322.
7. 854.
8. 770.
9. 315.
10. 504.
11. 1302.
12. 1155.
13. 864.
14. 1476.
15. 2193.
16. 810.
17. 3965.
18. 1224.
19. 5751.
20. 936.
21. 756.
22. 840.
23. 912.
24. 918.
25. 968.
26. 736.
27. 576.
28. 858.
29. 1024.
30. 1485.
31. 1496.
32. 1575.
33. 1806.
34. 2408.
35. 2860.
36. 3510.
37. 4212.
38. 4558.
39. 4428.
40. 3630.
41. 4464.
42. 5544.
43. 6656.
44. 6630.
45. 8208.
46. 9782.
47. 9620.
48. 9375.
49. 9512.
50. 8964.
51. 10,332.
52. 11,220.
53. 10,028.
54. 10,323.
55. 9776.
56. 9975.
57. 3870.
58. 4370.
59. 8262.
60. 11,396.
61. 15,167.
62. 22,104.
63. 11,433.
64. 15,580.
65. 20,709.
66. 28,980.
67. 10,998.
68. 16,560.
69. 22,050.
70. 31,360.
71. 5814.
72. 11,948.
73. 18,408.
74. 27,456.
75. 35,041.
76. 43,112.
77. 55,752.
78. 69,160.
79. 78,925.
80. 93,912.
81. 95,590.
82. 90,300.
83. 95,961.
84. 91,696.
85. 93,005.
86. 99,648.
87. 99,425.
88. 99,078.
89. 99,891.
90. 96,480.
91. 91,350.
92. 89,642.
93. 97,363.
94. 96,768.
95. 72,415.
96. 86,773.
97. 99,975.
98. 99,970.
99. 99,171.
100. 95,168.

Page 120.

1. 13.
2. 21.
3. 22.
4. 23.
5. 112.
6. 211.
7. 123.
8. 222.
9. 11.
10. 12.
11. 21.
12. 22.
13. 111.
14. 212.
15. 12.
16. 21.
17. 121.
18. 122.
19. 201.
20. 222.
21. 11.
22. 12.

23. 13.
24. 23.
25. 31.
26. 112.
27. 213.
28. 313.
29. 211.
30. 122.
31. 311.
32. 213.
33. 311.
34. 231.
35. 33.
36. 113.
37. 23.
38. 34.
39. 31.
40. 34.
41. 31.
42. 32.

Page 122.

1. 200 feet.
2. $20.70.
3. 11 sheep.
4. 900 inches.
5. 480 ounces.
6. 96 cents.
7. 16 pages.
8. 96 packages.
9. 25 gallons.
10. 7 ounces.
11. $50.
12. $100.
13. $175.
14. 15 miles.

Page 126.

1. $41\frac{3}{4}$.
2. $62\frac{5}{8}$.
3. $57\frac{1}{8}$.
4. $66\frac{3}{8}$.
5. $81\frac{7}{8}$.
6. $98\frac{7}{8}$.
7. $91\frac{3}{4}$.
8. $99\frac{5}{8}$.
9. $99\frac{3}{8}$.
10. $52\frac{5}{8}$.

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11. $42\frac{7}{8}$.
12. $46\frac{7}{8}$.
13. 78.
14. 46.
15. 19.
16. 40.
17. 73.
18. 95.
19. 89.
20. 80.
21. $3\frac{1}{2}$.
22. $2\frac{3}{4}$.
27. $3\frac{1}{2}$.
28. $18\frac{1}{2}$.
29. $35\frac{1}{3}$.
30. $53\frac{1}{2}$.
31. $88\frac{1}{3}$.
32. $7\frac{1}{4}$.
33. $37\frac{3}{8}$.
34. $15\frac{1}{4}$.
35. $21\frac{3}{8}$.
36. $27\frac{1}{2}$.
37. $25\frac{5}{8}$.
38. $36\frac{1}{4}$.
39. $8\frac{1}{3}$.
40. $39\frac{1}{4}$.
41. $17\frac{3}{8}$.
42. $37\frac{5}{8}$.
43. $15\frac{7}{8}$.
44. $45\frac{1}{3}$.
45. 47.
50. $4\frac{1}{3}$.

Page 128.

51. $916.61.
52. $778.91.
53. $1780.53\frac{1}{2}$.
54. $3431.48\frac{1}{2}$.
55. $76.11.
57. $657.66.
58. $4.17.
59. $76.50.
61. $58.20.
62. $457.12.
63. $977.67.
64. $963.69.
65. $725.04.
66. $31.25.
67. $.62.
68. $108.06.
69. $37.12\frac{1}{2}$.
70. $.34.

Page 130.

1. $4.14.
2. $22.50.
3. $14.
4. 21 cents.
5. $252.
6. $750.
7. 5 yards.
8. 96 jars.
9. 9 gallons.
10. 75 cents.
11. $1620.
12. $95.
13. $80.

Page 131.

14. 68 days.
15. 36 bushels.
16. $2.56.
17. $240.
18. 260 feet.
19. 60 miles.
20. 150 days.

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1. 31.
2. 31.
3. 24.
4. 31.
5. 42.
6. 23.
7. 41.
8. 11.
9. 21.
10. 21.
11. 11.
12. 21.
13. 11.
14. 21.
15. 31.
16. 41.
17. 23.
18. 31.
19. 111.
20. 111.
21. 321.
22. 322.
23. 300.
24. $302\frac{1}{15}$.

Page 132.

25. $20\frac{7}{24}$.
26. $40\frac{1}{21}$.
27. $50\frac{9}{11}$.
28. 203.
29. 202.
30. $202\frac{3}{14}$.
31. $202\frac{5}{14}$.
32. 101.
33. $101\frac{5}{15}$.
34. 203.
35. 304.
36. $200\frac{4}{21}$.

37. 304.
38. $430\frac{3}{11}$.
39. 203.
40. $431\frac{9}{11}$.
41. 202.
42. $120\frac{11}{61}$.
43. $221\frac{5}{11}$.
44. 123.
45. 325.
46. $231\frac{1}{11}$.
47. $101\frac{3}{5}$.
48. $34\frac{3}{11}$.
49. $122\frac{14}{11}$.
50. 103.
51. $20\frac{6}{13}$.

Page 137.

1. 47.
2. 84.
3. 32.
4. 73.
5. $81\frac{1}{4}$.
6. $68\frac{3}{4}$.
7. $38\frac{1}{2}$.
8. $20\frac{3}{8}$.
9. $70\frac{3}{8}$.
10. $83\frac{1}{4}$.
11. $6\frac{1}{4}$.
12. $18\frac{3}{8}$.
13. $19\frac{1}{2}$.
14. 17.
15. 50.
16. 26.
17. $31\frac{1}{2}$.
18. $13\frac{1}{8}$.
19. $10\frac{7}{8}$.
20. $22\frac{7}{8}$.
21. $2\frac{1}{4}$.
22. $13\frac{1}{8}$.
23. 21.
24. $30\frac{1}{8}$.
25. 39.
26. $1\frac{1}{8}$.
27. $10\frac{3}{8}$.
28. $10\frac{1}{2}$.
29. $10\frac{1}{2}$.
30. 10.
31. $9\frac{3}{4}$.
32. $20\frac{1}{2}$.
33. $41\frac{1}{4}$.
34. $18\frac{1}{2}$.
35. $26\frac{1}{4}$.
36. $31\frac{3}{8}$.
37. $67\frac{5}{8}$.
38. $19\frac{1}{2}$.
39. $1\frac{1}{8}$.
40. $19\frac{3}{8}$.

Page 138.

41. 185.
42. 48.
43. 203.
44. 19.
45. 90.
46. 29.
47. 70.
48. 4.
49. 428.
50. 17,376.
51. 1000.
52. 5600.
53. 5600.
54. 78.
55. 126.
56. 168.
57. 144.
58. 144.
59. 144.
60. 84.
61. 10.
62. 4410.
63. 15.
64. 15.
65. 13.
66. 200.
67. 48.
68. 60.
69. 32.
70. 60.

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1. $5.
2. 150 stamps.
3. 62 cows.
4. $16.
5. 28 pounds.

Page 139.

6. 144 pieces.
7. $12.80.
8. 40 boxes.
9. 49 inches.
10. 234 eggs.
11. $60.
12. 5 cents.
13. 15 cents.
14. $2.94.
15. 67 cents.
16. 7 packages.
17. $3.
18. 750 pounds.
19. 15 cents.
20. 46 cents.
21. 20 cents.

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22. 30 cents.
23. 5 yards.
24. 196 pounds.
25. 20 pieces.

---

1. 70,800.
2. 71,200.
3. 67,000.
4. 48,300.
5. 78,300.
6. 98,400.
7. 98,800.
8. 91,000.
9. 72,000.

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10. 90,000.
11. 88,800.
12. 84,150.
13. 95,000.
16. 77,400.
17. 83,700.
18. 89,100.
19. 93,000.
20. 67,200.
21. 95,370.
22. 99,540.
23. 81,480.
24. 88,480.

Page 143.

1. $7\frac{7}{8}$.
2. $9\frac{1}{2}$.
3. $16\frac{1}{2}$.
4. $19\frac{1}{2}$.
5. $20\frac{1}{8}$.
6. $19\frac{1}{8}$.
7. 48.
8. 37.
9. $22\frac{7}{8}$.
10. $45\frac{5}{8}$.
11. $84\frac{1}{2}$.
12. $89\frac{1}{2}$.
13. $11\frac{5}{8}$.
14. $39\frac{5}{8}$.
15. $48\frac{1}{8}$.
16. 7.
17. $22\frac{1}{2}$.
18. $19\frac{3}{8}$.

19. $30\frac{1}{2}$.
20. $36\frac{1}{2}$.
21. $35\frac{3}{8}$.
22. $25\frac{5}{8}$.
23. $12\frac{5}{8}$.
24. $4\frac{1}{8}$.
25. $97\frac{1}{2}$.
26. $15\frac{7}{8}$.
27. $91\frac{1}{8}$.
28. $31\frac{5}{8}$.
29. 43.
30. $88\frac{3}{8}$.
31. $9\frac{1}{8}$.
32. $21\frac{1}{8}$.
33. $7\frac{1}{8}$.
34. $22\frac{1}{2}$.
35. $5\frac{3}{8}$.
36. $11\frac{1}{8}$.
37. $15\frac{1}{8}$.
38. $14\frac{1}{8}$.
39. $16\frac{1}{2}$.
40. $17\frac{3}{8}$.
41. $19\frac{5}{8}$.
42. $28\frac{5}{8}$.
43. $38\frac{1}{8}$.
44. $47\frac{1}{8}$.
45. $50\frac{1}{8}$.
46. $8\frac{1}{2}$.
47. $28\frac{3}{8}$.
48. $21\frac{3}{4}$.
49. $7\frac{5}{8}$.

Page 145.

1. 54.
2. 54.
3. 54.
4. 54.
5. 44.
6. 32.
7. 24.
8. 33.
9. 32.
10. 45.
11. $45\frac{6}{24}$.
12. 45.
13. 43.
14. 56.
15. 65.
16. 78.
17. 81.
18. 86.
19. 82.
20. 66.
21. 72.
22. 88.
23. 123.
24. 138.
25. 77.
26. 133.
27. 37.
28. 77.
29. 115.
30. 97.
31. $85\frac{72}{84}$.
32. $97\frac{17}{85}$.
33. 92.
34. $65\frac{10}{88}$.
35. 68.
36. 95.
37. 143.
38. 541.
39. 328.
40. 216.
41. 304.
42. 271.
43. 74.
44. 143.
45. 206.
46. 184.
47. 136.
48. 108.
49. 204.
50. 162.
51. 143.
52. 152.
53. 84.
54. $143\frac{12}{66}$.
55. 123.
56. $135\frac{88}{66}$.
57. 96.
58. $121\frac{88}{77}$.
59. 126.
60. 103.
61. 109.
62. 112.
63. 107.
64. $97\frac{44}{88}$.
65. $89\frac{80}{99}$.
66. 54.
67. 97.
68. 102.
69. 92.
70. 350.
71. $232\frac{4}{36}$.
72. $108\frac{20}{45}$.
73. $174\frac{32}{54}$.
74. $139\frac{6}{63}$.
75. $136\frac{12}{72}$.
76. $119\frac{77}{81}$.
77. $131\frac{45}{64}$.
78. $86\frac{18}{36}$.
79. $120\frac{24}{48}$.
80. $206\frac{17}{34}$.
81. $304\frac{8}{28}$.
82. $206\frac{8}{35}$.
83. $234\frac{15}{42}$.
84. $178\frac{42}{42}$.
85. $116\frac{41}{63}$.
86. $120\frac{48}{72}$.
87. $400\frac{9}{24}$.
88. $545\frac{15}{18}$.
89. $555\frac{9}{18}$.
90. 355.
91. 91.
92. 142.
93. 113.
94. $112\frac{88}{88}$.
95. $87\frac{88}{88}$.
96. $51\frac{88}{88}$.
97. $103\frac{44}{88}$.
98. $102\frac{88}{88}$.
99. $103\frac{88}{77}$.

Page 146.

1. 136.
2. 32.
3. 34.
4. 122.
5. 87.
6. 75.
7. 24.
8. 65.
9. 25.
10. 49.

Page 147.

1. $136\frac{90}{300}$.
2. $93\frac{95}{400}$.
3. $119\frac{32}{500}$.
4. $107\frac{180}{600}$.
5. $89\frac{400}{700}$.
6. $90\frac{124}{800}$.
7. $92\frac{816}{900}$.
8. $80\frac{685}{1100}$.
9. $81\frac{1120}{1200}$.
10. $8\frac{602}{1300}$.

Page 149.

1. $27\frac{5}{9}$.
2. $47\frac{7}{9}$.
3. $51\frac{8}{9}$.
4. $38\frac{4}{9}$.
5. $66\frac{7}{9}$.
6. $99\frac{5}{9}$.
7. $68\frac{7}{9}$.
8. $94\frac{8}{9}$.
9. $95\frac{8}{9}$.
10. 85.
11. 99.
12. 23.
13. $44\frac{1}{9}$.
14. $70\frac{3}{9}$.
15. $27\frac{1}{9}$.
16. $42\frac{2}{9}$.
17. $53\frac{5}{9}$.
18. $98\frac{1}{9}$.
19. $83\frac{3}{9}$.
20. $35\frac{4}{9}$.
21. $6\frac{7}{9}$.
22. $23\frac{3}{9}$.
23. $22\frac{7}{9}$.
24. $31\frac{8}{9}$.
25. $31\frac{1}{9}$.
26. $12\frac{7}{9}$.
27. 10.
28. $27\frac{3}{9}$.
29. $16\frac{4}{9}$.
30. $53\frac{3}{9}$.
31. $3\frac{1}{9}$.
32. $26\frac{1}{9}$.
33. $21\frac{3}{9}$.
34. $4\frac{1}{9}$.
35. $44\frac{4}{9}$.
36. $11\frac{1}{9}$.
37. $8\frac{8}{9}$.
38. $16\frac{7}{9}$.
39. $24\frac{7}{9}$.
40. $32\frac{8}{9}$.
41. $40\frac{3}{9}$.
42. $48\frac{5}{9}$.
43. $56\frac{1}{9}$.
44. $64\frac{4}{9}$.
45. $72\frac{2}{9}$.
46. $79\frac{1}{9}$.
47. $27\frac{1}{9}$.
48. $17\frac{1}{9}$.
49. $29\frac{1}{9}$.
50. $62\frac{8}{9}$.

Page 151.

1. 99,684.
2. 85,731.
3. 95,772.
4. 94,770.
5. 94,095.
6. 89,622.
7. 96,882.
8. 95,914.
9. 99,507.
10. 91,344.
11. 86,592.
12. 97,020.
13. 93,832.
14. 79,328.
15. 91,464.
16. 96,266.
17. 94,520.
18. 94,518.
19. 90,750.
20. 93,396.
21. 96,170.
22. 97,908.
23. 89,159.
24. 87,472.
25. 97,768.
26. 95,918.
27. 43.
28. 78.
29. 32.
30. 24.
31. 14.
32. 13.
33. 14.
34. 12.
35. 11.
36. 9.

Page 152.

37. 8.
38. 13.
39. 15.
40. 13.
41. 24.
42. 23.
43. 45.
44. 75.
45. 33.
46. 22.
47. 8.
48. 4.
49. 6.
50. 4.
51. 68,580.
52. 96,621.
53. 96,859.
54. 96,740.
55. 92,640.
56. 85,181.
57. 82,926.
58. 64,684.
59. 76,020.
60. 97,768.
61. 90,752.
62. 70,455.
63. 98,049.
64. 86,592.
65. 98,245.
66. 71,604.
67. 99,770.
68. 98,802.
69. 81,804.
70. 95,081.
71. 98,245.
72. 98,245.
73. 92,486.
74. 75,072.

Page 153.

75. 88.
76. 105.
77. 99.
78. 98.
79. 69.
80. 69.
81. 168.
82. 95.
83. 90.
84. 93.
85. 186.
86. 154.
87. 232.
88. 368.
89. 297.
90. 100.
91. 102.
92. 205.
93. 255.

94. 320.
95. 456.
96. 675.
97. 880.
98. 615.

Page 154.

1. 360.
2. 1125.
3. 800.
4. 1200.
5. 1770.
6. 2800.
7. 2331.
8. 16,044.
9. 14,883.
10. 39,234.
11. 22,243.
12. 4400.

Page 155.

13. 6578.
14. 23,922.
15. 43,190.
16. 49,260.
17. 17,922.
18. 61,479.
19. 85,200.
20. 82,810.
21. 6888.
22. 13,552.
23. 39,528.
24. 51,968.
25. 14,610.
26. 25,280.
27. 50,904.
28. 65,400.
29. 84,252.
30. 96,560.
31. 79,380.
32. 30,537.
33. 42,372.
34. 107,028.
35. 96,444.
36. 92,376.
37. 337½.
38. 673⅜.
39. 1237½.
40. 1897½.
41. 1683½.
42. 2880¾.
43. 1869⅜.
44. 4575⅝.
45. 6286½.
46. 21,441¾.
47. 40,138.
48. 44,500⅘.
49. 42,274⅞.
50. 99,682⅔.
51. 65,166⅔.
52. 4231.
53. 3152.
54. 2405.
55. 1600.
56. 1623.
57. 1405.
58. 1234.
59. 1035.
60. 2305.
61. 2046.
62. 1653.
63. 1408.
64. 1305.
65. 1060.
66. 1003.
67. 3265.
68. 907.
69. 807.
70. 486.
71. 325.
72. 247.
73. 189.
74. 155.
75. 138.
76. 123.
77. 109.
78. 406.
79. 308.
80. 203.
81. 170.
82. 146.
83. 123.
84. 105.
85. 104.
86. 98.
87. 48.
88. $33\frac{80}{3012}$.
89. 24.
90. 19.
91. 16.
92. 14.
93. 12.
94. 11.
95. 98.
96. 87.
97. 75.
98. 33.
99. 23.
100. 9.
101. 4.
102. 8.

Page 158.

1. $7.
2. $2.82.
3. 192 pints.
4. 20 yards.
5. $2.40.
6. 40 cents.

Page 159.

7. $3.
8. $2.25.
9. 2½ yards.
10. 108 quarters.
11. $72.50.
12. 3 cents.
13. 86 feet.
14. $7.80.
15. $2.40.
16. 95 cents.
17. 99 cents.
18. 48 eggs; 144 eggs.
19. 40 bushels.
20. $60.
21. 413 butterflies.
22. 99 cents.

Page 160.

23. 130 yards.
24. 7¼ acres.
25. $1.75.

---

1. 91,448.
2. 86,400.
3. 97,886.
4. 90,288.
5. 89,415.
6. 88,971.
7. 89,208.
8. 82,766.
9. 99,696.
10. 73,140.
11. 82,602.
12. 99,960.
13. 97,633.
14. 96,348.
15. 180.
16. 232.
17. 348.
18. 567.
19. 864.

20. 1120.
21. 777.
22. 945.
23. 1100.
24. 1343.
25. 496.
26. $1454\frac{2}{5}$.
27. 96,000.
28. 99,712.
29. 96,888.
30. 99,328.
31. 77,608.
32. 99,450.
33. 99,902.
34. 95,841.
35. 61,845.
36. 99,102.
37. 96,696.
38. 92,976.
39. 99,051.
40. 93,345.
41. 96,744.
42. 88,920.
43. 99,601.
44. 99,485.
45. 2000.
46. 2800.
47. 24,000.
48. 24,500.
49. 99,000.
50. 96,000.
51. 81,081.

Page 161.

1. $129\frac{1}{4}$.
2. $92\frac{7}{8}$.
3. $79\frac{7}{8}$.
4. $97\frac{3}{8}$.
5. $69\frac{1}{6}$.
6. $27\frac{5}{6}$.
7. $47\frac{5}{8}$.
8. $99\frac{3}{8}$.
9. $82\frac{5}{6}$.
10. $108\frac{1}{4}$.
11. $64\frac{5}{8}$.
12. 91.
13. 95.
14. $37\frac{7}{9}$.
15. 26.

Page 162.

16. 86.
17. $81\frac{1}{3}$.
18. 49.
19. $83\frac{1}{3}$.
20. $87\frac{1}{2}$.
21. $64\frac{1}{3}$.
22. $28\frac{1}{3}$.
23. $37\frac{1}{2}$.
24. $35\frac{1}{3}$.
25. $54\frac{1}{4}$.
26. $69\frac{3}{8}$.
27. $30\frac{5}{8}$.
28. $81\frac{1}{3}$.
29. $\frac{1}{8}$.
30. $75\frac{1}{8}$.
31. $8\frac{3}{8}$.
32. $6\frac{1}{2}$.
33. $18\frac{1}{2}$.
34. $43\frac{3}{8}$.
35. $12\frac{3}{4}$.
36. $11\frac{5}{8}$.
37. $24\frac{7}{8}$.
38. $18\frac{3}{8}$.
39. $27\frac{7}{8}$.
40. $40\frac{3}{8}$.

---

1. $2857\frac{3}{21}$.
2. $3134\frac{14}{31}$.
3. $1225\frac{14}{41}$.
4. $1622\frac{45}{51}$.
5. $990\frac{6}{61}$.
6. $5387\frac{10}{71}$.
7. $7137\frac{18}{81}$.
8. $2822\frac{22}{91}$.
9. $1636\frac{18}{33}$.
10. $1966\frac{15}{25}$.
11. $811\frac{5}{52}$.
12. $787\frac{27}{45}$.
13. $478\frac{37}{74}$.
14. $222\frac{12}{62}$.
15. $279\frac{24}{42}$.
16. $711\frac{77}{85}$.
17. $182\frac{25}{102}$.
18. $432\frac{205}{207}$.
19. $153\frac{184}{101}$.
20. $86\frac{80}{408}$.
21. $181\frac{81}{503}$.
22. $113\frac{168}{804}$.
23. $104\frac{179}{705}$.
24. $111\frac{187}{809}$.
25. $70\frac{219}{806}$.
26. $709\frac{4}{112}$.
27. $219\frac{89}{219}$.
28. $132\frac{81}{318}$.
29. $42\frac{177}{417}$.
30. $182\frac{425}{518}$.
31. $157\frac{188}{615}$.
32. $49\frac{375}{714}$.
33. $25\frac{678}{813}$.
34. $40\frac{58}{912}$.
35. $30\frac{88}{1009}$.
36. $38\frac{232}{2008}$.
37. $5\frac{595}{3007}$.
38. $12\frac{3168}{1008}$.
39. $19\frac{2497}{5005}$.
40. $14\frac{7777}{6004}$.
41. $2\frac{3578}{7003}$.
42. $7\frac{759}{8002}$.
43. $3\frac{8444}{9001}$.
44. $17\frac{5228}{12234}$.
45. $3\frac{1}{13235}$.
46. $4\frac{2}{23174}$.
47. 6.
48. 3.

Page 164.

1. 31 barrels.
2. 6 yards.
3. 380 inches.

Page 165.

4. $1\frac{1}{2}$ yards.
5. 25 cents; $1.
6. 16 cents.
7. 98 cents.
8. $3\frac{1}{2}$ pounds.
9. 39 pints.
10. 145 sheep.
11. $2.
12. 93 cents.
13. 6 weeks.
14. 35 gallons.
15. 10 lb. 5 oz.
16. $2.

Page 166.

17. 41 pounds.
18. $3.60.
19. 150 days; 3 days.
20. $2.05.
21. $39.
22. $225.
23. $2.25.
24. $8.22.
25. $40.

Page 169.

1. 603,275.
2. 678,456.
3. 759,795.
4. 641,426.
5. $2714.42.

6. $8502.43.
7. $7269.80.
8. $9885.02.
9. 300,424.
10. 913,092.
11. $220,119.
12. $1912.09.
13. $359,809.
14. 414,867.
15. $161,715.
16. 173,929.
17. $2952.51.
18. 399,952.
19. $1624.43.

Page 170.

20. 868,980.
21. 895,048.
22. 954,048.
23. 996,450.
24. 592,320.
25. 864,128.
26. 970,485.
27. 940,215.
28. 967,890.
29. 954,087.
30. 906,205.
31. 968,464.
32. 886,730.
33. 864,565.
34. 941,408.
35. 948,708.
36. 972,930.
37. 761,472.
38. 955,320.
39. 969,855.
40. 976,372.
41. 926,328.
42. 925,245.
43. 856,674.
44. 977,724.
45. 963,976.
46. 887,112.
47. 629,405.
48. 890,765.
49. 933,725.
50. 2123.
51. 1203.
52. 1303.
53. 1203.
54. 1031.
55. 2402.
56. 3002.
57. 3030.
58. 10,444.
59. 1060.
60. 1011.
61. 1012.
62. 1013.
63. 1011.
64. 1101.
65. 1102.
66. 220.
67. 303.
68. 150.
69. 606.
70. 222.
71. $300\frac{24}{25}\frac{38}{72}$.
72. $306\frac{20}{30}\frac{18}{06}$.
73. 219.
74. $101\frac{46}{50}\frac{83}{41}$.
75. $154\frac{52}{60}\frac{85}{36}$.
76. $112\frac{53}{71}\frac{78}{02}$.
77. $112\frac{15}{81}\frac{54}{03}$.
78. $861\frac{8}{22}$.
79. $833\frac{8}{24}$.
80. $903\frac{8}{26}$.
81. $982\frac{8}{28}$.
82. $1313\frac{12}{32}$.
83. $2196\frac{32}{35}$.
84. $2218\frac{34}{38}$.
85. $2279\frac{12}{44}$.
86. $3134\frac{44}{45}$.
87. $3142\frac{9}{55}$.
88. $3009\frac{8}{63}$.
89. $3034\frac{56}{66}$.
90. $3050\frac{67}{69}$.
91. $3071\frac{4}{75}$.
92. $2016\frac{16}{124}$.
93. $1234\frac{147}{256}$.
94. $1132\frac{288}{361}$.
95. $1355\frac{52}{425}$.
96. 504.
97. 306.
98. 203.
99. 105.
100. $109\frac{1147}{1023}$.

Page 171.

101. $59\frac{1}{4}$.
102. $19\frac{1}{2}$.
103. 9639.
104. 12,141.
105. 96.
106. 96.
107. $12\frac{1}{4}$.
108. $60\frac{3}{4}$.
109. $61\frac{1}{4}$.
110. 300.
111. 300.
112. 2.
113. 8.
114. 78.
115. 162.
116. 231.
117. 36.
118. 648.
119. $46\frac{1}{2}$.
120. $70\frac{3}{4}$.
121. 12,126.
122. 187,440.
123. 68.
124. 975.
125. 34.
126. 125.
127. 5.
128. 138.
129. 150.
130. 78.

Page 172.

1. 760 yards.
2. 240 hf. pt.
3. $3.45.
4. $248.
5. $210.
6. $4\frac{3}{4}$ pounds.
7. $41\frac{1}{3}$ bushels.
8. $4.98.

Page 173.

9. $1\frac{1}{2}$ minutes.
10. 1440 matches.

---

1. $5\frac{5}{8}$.
2. $12\frac{1}{2}$.
3. $15\frac{3}{5}$.
4. 20.
5. $27\frac{1}{3}$.
6. $42\frac{5}{8}$.
7. $38\frac{1}{3}$.
8. $56\frac{1}{4}$.
9. $134\frac{1}{3}$.
10. $134\frac{7}{8}$.
11. $88\frac{1}{2}$.
12. 50.
13. $44\frac{1}{2}$.
14. $26\frac{2}{3}$.
15. $36\frac{3}{4}$.
16. $9\frac{5}{8}$.
17. $46\frac{7}{8}$.
18. $147\frac{1}{3}$.
19. $37\frac{1}{2}$.
20. $73\frac{1}{3}$.

21. $7\frac{1}{4}$.
22. $4\frac{1}{4}$.
23. $86\frac{3}{8}$.
24. $29\frac{1}{8}$.
25. $34\frac{1}{8}$.
26. $59\frac{1}{8}$.
27. $61\frac{1}{8}$.
28. $19\frac{1}{3}$.
29. $17\frac{1}{3}$.
30. $7\frac{1}{3}$.
31. $8\frac{1}{3}$.
32. $18\frac{1}{3}$.
33. $49\frac{1}{2}$.
34. $25\frac{1}{2}$.
35. $74\frac{1}{3}$.
36. $23\frac{1}{2}$.

Page 174.

1. 46,512.
2. 144,536.
3. 253,840.
4. 132,435.
5. 306,130.
6. 354,488.
7. 87,084.
8. 199,014.
9. 784,770.
10. 934,164.
11. 784,770.
12. 934,164.
13. 30,504.
14. 54,756.
15. 37,260.
16. 138,624.
17. 616,302.
18. 104,148.
19. 805,460.
20. 93,912.
21. 151,782.
22. 548,730.
23. 846,300.
24. 969,600.
25. 617,120.
26. 434,420.
27. 47,196.
28. 47,272.
29. 47,082.
30. 137,598.
31. 59,660.
32. 59,508.
33. 137,427.
34. 78,150.
35. 209,664.
36. 844,662.
37. 979,016.
38. 998,016.
39. 17,329$\frac{1}{3}$.
40. 58,867$\frac{1}{3}$.
41. 760,249.
42. 369,123.
43. 17,367$\frac{5}{7}$.
44. 30,250.
45. 850,950.
46. 95,482$\frac{4}{5}$.
47. 938,475.
48. 935,712.
49. 781,137$\frac{1}{2}$.
50. 954,320.

Page 176.

1. $20\frac{7}{10}$.
2. $25\frac{9}{10}$.
3. $39\frac{1}{10}$.
4. $70\frac{3}{10}$.
5. 81.
6. $9\frac{3}{10}$.
7. $21\frac{1}{10}$.
8. $7\frac{1}{10}$.
9. $92\frac{3}{10}$.
10. $57\frac{3}{5}$.

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11. $\frac{9}{20}$.
12. $2\frac{9}{20}$.
13. $4\frac{9}{20}$.
14. $4\frac{13}{20}$.
15. $18\frac{13}{20}$.
16. $33\frac{13}{20}$.
17. $53\frac{17}{20}$.
18. 76.
19. 105.
20. $48\frac{1}{20}$.
21. $9\frac{1}{20}$.
22. $61\frac{3}{20}$.
23. $63\frac{7}{20}$.
24. $7\frac{11}{20}$.
25. $15\frac{1}{20}$.
26. $32\frac{7}{10}$.
27. $29\frac{13}{20}$.
28. $29\frac{19}{20}$.
29. 82.
30. $82\frac{7}{8}$.
31. $21\frac{1}{2}$.
32. $31\frac{3}{4}$.
33. $41\frac{1}{4}$.
34. $51\frac{3}{5}$.
35. $61\frac{5}{6}$.
36. $61\frac{7}{8}$.
37. $71\frac{5}{6}$.
38. $84\frac{1}{8}$.
39. $79\frac{1}{12}$.
40. $65\frac{1}{20}$.
41. $59\frac{1}{3}$.
42. $70\frac{1}{8}$.
43. $38\frac{3}{10}$.
44. $7\frac{1}{4}$.
45. $9\frac{3}{8}$.
46. $8\frac{3}{15}$.
47. $78\frac{1}{3}$.
48. $55\frac{1}{12}$.
49. $47\frac{3}{20}$.
50. $39\frac{3}{10}$.

Page 179.

1. 3210.
2. 4321.
3. 765.
4. $3450\frac{1}{154}$.
5. 5403.
6. $4506\frac{1}{156}$.
7. 6063.
8. 7006.
9. 6003.
10. 6005.
11. 7001.
12. 5203.
13. $6715\frac{60}{135}$.
14. 5701.
15. $1020\frac{92}{153}$.
16. 2034.
17. 3240.
18. 4003.
19. $5041\frac{1}{133}$.
20. $4774\frac{52}{194}$.
21. $1789\frac{1}{2}\frac{1}{15}$.
22. $1509\frac{80}{252}$.
23. $1155\frac{1}{2}\frac{17}{16}$.
24. $2631\frac{1}{2}\frac{67}{77}$.
25. $2347\frac{1}{2}\frac{65}{95}$.
26. $2981\frac{2}{3}\frac{14}{18}$.
27. $1435\frac{16}{347}$.
28. $499\frac{14}{375}$.
29. $1545\frac{3}{8}\frac{5}{56}$.
30. $720\frac{60}{424}$.

31. $2117\frac{19}{45}$.
32. $1707\frac{344}{415}$.
33. $1607\frac{251}{351}$.
34. $1615\frac{400}{462}$.
35. $1191\frac{448}{504}$.
36. $1053\frac{447}{844}$.
37. $1008\frac{223}{689}$.
38. $879\frac{504}{724}$.
39. $990\frac{714}{1417}$.
40. $600\frac{40}{961}$.
41. $608\frac{196}{1034}$.
42. $461\frac{106}{2165}$.
43. $307\frac{489}{3254}$.
44. $185\frac{1574}{4318}$.
45. $153\frac{4758}{5409}$.
46. $25\frac{3248}{6521}$.
47. $70\frac{3284}{7611}$.
48. $30\frac{4575}{8794}$.
49. $32\frac{8312}{9801}$.
50. $283\frac{1445}{1757}$.
51. $230\frac{603}{2043}$.
52. $251\frac{2513}{3087}$.

Page 181.

1. 240 bushels.
2. 58 cents.
3. 5 cows.
4. 90 cents.
5. $5500.
6. $226\frac{1}{3}$ acres.
7. $5.88.
8. $90.
9. 402.
10. 4 cows.
11. 16 days.
12. 3 yards.
13. 80 quarts.

Page 185.

1. $18\frac{7}{12}$.
2. $24\frac{7}{12}$.
3. $45\frac{11}{12}$.
4. $66\frac{1}{12}$.
5. $79\frac{5}{12}$.
6. $75\frac{1}{12}$.
7. $17\frac{5}{12}$.
8. $16\frac{5}{12}$.
9. $49\frac{5}{12}$.
10. $38\frac{1}{12}$.
11. $18\frac{1}{12}$.
12. $42\frac{5}{12}$.
13. $65\frac{7}{12}$.
14. $29\frac{11}{12}$.
15. $75\frac{11}{12}$.
16. $116\frac{5}{8}$.
17. $50\frac{7}{12}$.
18. $65\frac{1}{3}$.
19. $92\frac{3}{4}$.
20. $97\frac{1}{12}$.
21. $28\frac{7}{8}$.
22. 59.
23. $12\frac{7}{8}$.
24. $99\frac{1}{12}$.
25. $96\frac{3}{4}$.
26. $25\frac{1}{3}$.
27. $71\frac{1}{8}$.
28. $56\frac{1}{8}$.
29. $25\frac{1}{12}$.
30. $3\frac{5}{12}$.
31. $14\frac{1}{4}$.
32. $59\frac{1}{3}$.
33. $38\frac{3}{4}$.
34. $31\frac{3}{8}$.
35. $18\frac{1}{2}$.
36. $31\frac{1}{2}$.
37. $30\frac{3}{8}$.
38. $22\frac{3}{4}$.
39. $41\frac{5}{8}$.
40. $37\frac{7}{8}$.
41. $13\frac{5}{8}$.
42. $3\frac{1}{3}$.
43. $27\frac{1}{4}$.
44. $31\frac{1}{8}$.
45. $13\frac{1}{2}$.
46. $32\frac{3}{8}$.
47. $13\frac{1}{8}$.
48. $14\frac{1}{3}$.
49. $41\frac{1}{8}$.
50. $20\frac{1}{12}$.

Page 186.

1. 117 ounces.
2. 4 lb. 5 oz.
3. 20 gal. 2 qt.
4. 59 quarts.
5. 23 qt. 1 pt.
6. 57 pints.
7. 75 pecks.
8. 143 quarts.
9. 12 pk. 1 qt.
10. 21 bu. 3 pk.
11. 1568 quarts.
12. 180 inches.
13. 44 feet.
14. 159 inches.
15. 9 ft. 11 in.
16. 23 yd. 1 ft.
17. 44 pounds.
18. 88 gallons.
19. 65 quarts.
20. 151 bushels.
21. 43 pecks.
22. 85 ft. 3 in.
23. 60 bu. 3 pk.
24. 13 ft. 1 in.
25. 67 gal. 2 qt.

Page 188.

1. 80.
2. $1200; $240.
3. $1.

Page 189.

4. $4.
5. 29 tons.
6. 195 days.
7. 13 cents.
8. 416 yards.
9. $405.
10. 537 pounds.
11. 35 plants.
12. 341 passengers.
13. $3000.
14. Lost $20.
15. 1799.
16. 8 years.

Page 190.

17. $3.
18. $420.
19. $225.
20. 46 boys.
21. $2.
22. $216.
23. 10 cents.
24. 9 months.
25. 200 eggs.

---

1. 835,539.
2. 759,645.
3. 888,732.
4. 869,649.
5. 805,050.
6. 746,108.
7. 902,000.
8. 963,214.
9. 855,922.
10. 957,032.
11. 704,175.
12. 593,164.
13. 986,592.
14. 962,304.
15. 943,114.

16. 831,875.
17. 833,316.
18. 505,134.

Page 191.

19. $190\frac{2}{3}$.
20. 500.
21. 420.
22. 1845.
23. 987.
24. 1071.
25. 1612.
26. 1645.
27. 2583.
28. 3885.
29. 4100.
30. 780,096.
31. 991,782.
32. 943,260.
33. 984,328.
34. 892,320.
35. 952,408.
36. 933,450.
37. 875,706.
38. 952,714.
39. 970,169.
40. 954,530.
41. 3519.
42. 3616.
43. 6132.
44. 4557.
45. 9568.
46. 10,791.
47. 17,572.
48. 39,333.
49. 76,775.
50. 97,460.
51. 69,000.
52. $2218\frac{2}{29}$.
53. $786\frac{1}{3}$.
54. $1618\frac{1}{10}$.
55. $1046\frac{8}{33}$.
56. $1033\frac{1}{3}$.
57. $841\frac{3}{7}$.
58. $609\frac{3}{5}$.
59. $215\frac{4}{5}$.
60. $223\frac{81}{191}$.
61. $260\frac{2}{3}$.
62. $89\frac{3}{5}$.
63. $83\frac{1}{4}$.
64. $40\frac{2}{5}$.
65. $78\frac{4}{6}$.
66. $81\frac{3}{7}$.
67. $32\frac{4}{5}$.
68. $99\frac{4}{5}$.
69. $43\frac{3}{5}$.
70. $260\frac{4}{1}$.
71. $233\frac{5}{19}$.
72. $281\frac{2}{2}$.
73. $179\frac{2}{3}$.
74. $166\frac{5}{4}$.
75. $51\frac{2}{5}$.
76. $107\frac{1}{6}$.
77. $20\frac{3}{7}$.
78. $50\frac{2}{8}$.
79. $90\frac{3}{5}$.
80. $579\frac{1}{2}$.
81. $2332\frac{3}{4}$.
82. $767\frac{1}{4}$.
83. $628\frac{3}{5}$.
84. $1398\frac{4}{8}$.
85. $1021\frac{7}{8}$.
86. $1051\frac{3}{8}$.
87. $974\frac{7}{8}$.
88. $108\frac{1}{1}$.
89. $278\frac{2}{2}$.
90. $84\frac{1}{3}$.
91. $184\frac{1}{4}$.
92. $905\frac{4}{5}$.
93. $554\frac{2}{6}$.
94. $951\frac{1}{7}$.
95. $285\frac{2}{8}$.
96. $144\frac{3}{5}$.
97. $821\frac{7}{7}$.
98. $91\frac{9}{1}$.
99. $241\frac{2}{2}$.
100. $237\frac{2}{3}$.
101. $63\frac{8}{4}$.
102. $181\frac{4}{5}$.

Page 192.

103. 97.
104. 48.
105. 32.
106. 24.
107. 18.
108. 16.
109. 14.
110. 12.
111. 11.
112. 44.
113. 33.
114. 22.

Page 193.

1. $1\frac{8}{15}$.
2. $4\frac{8}{15}$.
3. $7\frac{8}{15}$.
4. $16\frac{1}{15}$.
5. $37\frac{8}{15}$.
6. $13\frac{2}{15}$.
7. $11\frac{7}{15}$.
8. $31\frac{4}{15}$.
9. $8\frac{1}{15}$.
10. $71\frac{2}{15}$.
11. $18\frac{1}{15}$.
12. $26\frac{8}{15}$.
13. $53\frac{1}{5}$.
14. $99\frac{1}{3}$.
15. $91\frac{1}{3}$.
16. $47\frac{5}{12}$.
17. $77\frac{1}{15}$.
18. $86\frac{8}{15}$.
19. $51\frac{1}{3}$.
20. $63\frac{1}{3}$.
21. $88\frac{7}{15}$.
22. $93\frac{5}{8}$.
23. $94\frac{3}{4}$.
24. $93\frac{7}{8}$.
25. $81\frac{1}{12}$.
26. $8\frac{1}{2}$.
27. $76\frac{1}{12}$.
28. $88\frac{5}{8}$.
29. $95\frac{9}{20}$.
30. $93\frac{1}{12}$.
31. $71\frac{1}{3}$.
32. $3\frac{2}{3}$.
33. $24\frac{3}{5}$.
34. $18\frac{4}{5}$.
35. $21\frac{4}{5}$.
36. $43\frac{1}{12}$.
37. $44\frac{2}{15}$.
38. $19\frac{1}{2}$.
39. $14\frac{7}{12}$.
40. $35\frac{1}{12}$.

Page 195.

1. 260,512.
2. 110,124.
3. 387,024.
4. 237,394.
5. 505,580.
6. 594,672.
7. 531,696.
8. 178,654.
9. 194,508.
10. 177,045.
11. 329,141.
12. 537,966.

Page 196.

13. 503,036.
14. 354,585.
15. 348,087.
16. 781,529.
17. 75,854.
18. 63,616⅔.
19. 56,818½.
20. 80,647⅓.
21. 77,371⅛.
22. 114,608⅘.
23. 220,676.
24. 313,985½.
25. 434,661⅜.
26. 447,673.
27. 488,748.
28. 551,536⅞.
29. 486,029⅜.
30. 881,001½.
31. 101,376.
32. 162,582.
33. 145,116.
34. 130,245.
35. 84,632.
36. 577,771.
37. 194,142.
38. 806,922.
39. 834,725.
40. 696,822.
41. 92,501½.
42. 205,979.
43. 301,058.
44. 293,336⅔.
45. 397,087.
46. 564,389⅘.
47. 378,670½.
48. 489,303⅓.
49. 571,693⅓.
50. 352,315½.
51. $3646\frac{12}{17}$.
52. $2376\frac{13}{37}$.
53. $1002\frac{33}{47}$.
54. $1578\frac{44}{57}$.
55. $326\frac{49}{67}$.
56. $711\frac{7}{77}$.
57. $361\frac{19}{87}$.
58. $441\frac{7}{97}$.
59. $3297\frac{21}{28}$.
60. $977\frac{14}{38}$.
61. $2159\frac{14}{48}$.
62. $311\frac{25}{58}$.
63. $1157\frac{8}{68}$.
64. $1073\frac{4}{78}$.
65. $1048\frac{11}{88}$.
66. $3144\frac{8}{98}$.
67. $631\frac{54}{178}$.
68. $172\frac{87}{272}$.

Page 197.

69. $59\frac{156}{478}$.
70. $18\frac{147}{874}$.
71. $81\frac{328}{877}$.
72. $123\frac{662}{778}$.
73. $21\frac{715}{879}$.
74. $33\frac{158}{378}$.
75. $3223\frac{158}{268}$.
76. $370\frac{44}{368}$.
77. $410\frac{138}{467}$.
78. $908\frac{242}{268}$.
79. $930\frac{182}{278}$.
80. $460\frac{780}{764}$.
81. $417\frac{255}{685}$.
82. $263\frac{402}{868}$.
83. $255\frac{1322}{2758}$.
84. $197\frac{1328}{3725}$.
85. $194\frac{2518}{4758}$.
86. $116\frac{2449}{5747}$.
87. $54\frac{2582}{7774}$.
88. $68\frac{1473}{7765}$.
89. $29\frac{2587}{8974}$.
90. $76\frac{4628}{8783}$.
91. $135\frac{1150}{2659}$.
92. $228\frac{1188}{3668}$.
93. $29\frac{2425}{4674}$.
94. $119\frac{1323}{5665}$.
95. $47\frac{2025}{6858}$.
96. $97\frac{2241}{7841}$.
97. $50\frac{2058}{7858}$.
98. $63\frac{423}{962}$.
99. $7\frac{6243}{27387}$.
100. $5\frac{2510}{37257}$.

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